NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja
FACULTY OF SCIENCES
September 2020_1 Examination

Course Code: MTH 301
Course Title: Functional Analysis
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Number One (1) And Any Other Four (4) Questions

1. (a) Explain what is meant by a topology τ on a non-empty set X. **(3 marks)**
   (b) Give an example of discrete and indiscrete topology. **(4 marks)**
   (c) Let X be a complete metric space and $\{O_n\}$ be a countable collection of dense open subsets of X. Show that $\bigcup O_n$ is not empty. **(10 marks)**
   (d)) Let K ⊆ X be compact. Show that K is bounded. **(5 marks)**

2. (a) The collection Zd defined as Zd = {A ⊆ X : x ∈ A implies there exists r > 0 such that B(x, r) ⊆ A} is a topology on X, known as the topology induced by the given metric d. In a metric space (X, d) for each x ∈ X, r > 0, show that B(x, r) is an open subset of (X, Zd). **(5 marks)**
   (b) Let K be a collection of nonempty closed subsets of a compact space T such that every finite subcollection of K has a nonempty intersection. Show that the intersection of all sets from K is non-empty. **(7 marks)**

3. (a) State Heine-Borel theorem. **(2 marks)**
   (b) Show that a continuous image of a compact space is compact. **(10 marks)**

4. (a) State axioms of addition of a real number system $(\Re, +, \cdot)$ **(4 marks)**
   (b) Prove that a subspace T of a topological space S is disconnected iff it is separated by some open subsets U, V of S. **(8 marks)**

5. Let (X, d) and (Y, d_1) be metric spaces and g is a mapping of X into Y. Let τ and τ_1 be the topologies determined by d and d_1 respectively. Show that g :(x, τ) →(y, τ) is continuous if and only if $x_n \to x \Rightarrow g(x_n) \to g(x)$: that is if $x_1, x_2, \ldots x_n, \ldots$ is a sequence of points in (X, d) converging to x, then the sequence of points $g(x_1), g(x_2), \ldots g(x_n), \ldots$ in (Y, d) converges to g(x). **(12 marks)**

6. Prove that a set C is a closed set if and only if it contains all its limit points **(12 marks)**