NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
September 2020_1 Examination

Course Code: **MTH 303**
Course Title: **Vectors and Tensors Analysis**
Credit Unit: **3**
Time Allowed: **3 Hours**
Instruction: **Answer Question Number One and Any Other Four Questions**

1. (a) (i) Show that the angle between two vectors $\underline{a} = (a_1, a_2, a_3)$ and $\underline{b} = (b_1, b_2, b_3)$ can be expressed as $\theta = \cos^{-1}\frac{a_1b_1 + a_2b_2 + a_3b_3}{|\underline{a}||\underline{b}|}$ **(4 marks)**

   (ii) Give the definition of the magnitude of a vector product in terms of the area of a parallelogram. **(2 marks)**

   (iii) Show that if $\underline{a} \wedge \underline{b} = 0$ and neither $\underline{a} = 0$ nor $\underline{b} = 0$, then $\underline{a}$ and $\underline{b}$ are parallel.

   (b) Define the followings: **(3 marks)**

   (i) Derivative of a vector function $\underline{A}(u)$ **(2 marks)**

   (ii) Gradient of the function $\emptyset(x, y, z)$ **(2 marks)**

   (iii) the curl of a vector function $\underline{A}$ **(3 marks)**

   (c) (i)State the Divergence Theorem **(3 marks)**

   (ii) State Stoke's Theorem **(3 marks)**

2. (a) If $\underline{A}, \underline{B}, \underline{C}$ are differentiable vector functions of a scalar u, give the expressions for the following derivatives. I. $\frac{d}{du}\left(\underline{A} \cdot \left(\underline{B} \wedge \underline{C}\right)\right)$ II. $\frac{d}{du}\left(\underline{A} \wedge \left(\underline{B} \wedge \underline{C}\right)\right)$ **(4 marks)**

   (b) Given a vector $\underline{Q} = \cos 3ti + \sin 3tj$, obtain $\left|\frac{d\underline{Q}}{dt}\right|$ **(4 marks)**

   (c) Show that Grad $\emptyset \cdot dr = d\emptyset$ **(4 marks)**

3. (a) Show that $\frac{d\emptyset}{ds} = \frac{dr}{ds} \cdot Grad\ \emptyset$ **(4 marks)**

(b) If $\emptyset(n, y, z) = 2n^2yz^2$, find $\nabla\emptyset$ **(4 marks)**

(c) If $\emptyset = n^2yz$ a scalar function and $\underline{A} = 2nzi + yzj - ny^2k$, find $\nabla \cdot (\emptyset\underline{A})$ at the point (1, -1, 1). **(4 marks)**

4. (a) What is summation convention? **(3 marks)**

(b) Define the contravariant component of a tensor of the second rank. **(3 marks)**

(c) Explain the terms 'Outer product of tensors' and 'Contraction' **(6 marks)**

5. (a) Evaluate $\int_c (x + 3y)dx$ from A(0, 1) to B(2, 5) along the curve $y = 1+x^2$. **(6 marks)**

(b) Evaluate $I = \int_c \{(x^2 + 2y)dx + xydy\}$ from O(0, 0) to B(1, 4) along the curve $y = 4x^2$. **(6 marks)**

6. (a) Evaluate the double integral $\iint_R (x^2 - y^3)dxdy$, $1 \le x \le 2,\ 2 \le y \le 4$ **(6 marks)**

(b) A solid is enclosed by the planes z = 0, y = 1, y = 2, x = 0, x = 3 and the surface $z = x + y^2$. Determine the volume of the solid so formed **(6 marks)**