NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2020_1 EXAMINATIONS

Course Code: MTH 304
Course Title: Complex Analysis I
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Question Number One and Any other Four Questions.

1. a) If $f(z) = u(x,y) + iv(x,y)$ is analytic in a region $\Re$ and $u(x,y) = y^3 - 3x^2 y$.
Find $v(x,y)$? **[5 Marks]**

b) Show that (i) $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$. (ii) $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$ **[3 Marks]**

c) Evaluate $\int_{3i}^{2+4i} (2y + x^2)\,dx + (3x - y)\,dy$ along the parabola $x = 2t$ and $y = t^2 + 3$. **[5 Marks]**

d) Let $w = f(z) = z^2$. Find the values of w which correspond to:
(i) $z = -2 + i$ **[2½ Marks]**
(ii) $z = 1 - 3i$ **[2½ Marks]**

e) Express in polar form the complex number $z = -3i$ **[4 Marks]**

2. a) Given the complex function $f(z) = \dfrac{1}{(z^2 + 4)}$. Find the first four terms of the Taylor series expansion $f(z)$ about $z = -i$. **[7 Marks]**

b) Find $\dfrac{df}{dz}$ of this function: $f(z) = 4x + y + i(-x + 4y)$ along real axis. **[5 Marks]**

3. a) Using Cauchy – Riemann equation, show that $f(z) = z^3$ is analytic in the entire z – plane **[8 Marks]**

b) Find $f(z)$ such that $f^{I}(z) = 4z - 3$ and $f(1+i) = -3i$ **[4 Marks]**

4. a) If $z_1 = 2 + i$ and $z_2 = 3 - 2i$.

(i) Evaluate $|3z_1 - 4z_2|$ **[4 Marks]**

(ii) Find the dot product of $z_1 \bullet z_2$. **[3 Marks]**

b) Find the value of the integral $\int_C (x+y)dx + x^2 y dy$ along $y = x^2$,

having (0, 0) and (3, 9) as end points. **[5 Marks]**

5. a) Show that the function $e^x(\cos y + i\sin y)$ is an analytic function, find its derivative. **[5 Marks]**

b) Evaluate (i) $\lim_{z\to 1+i}(z^2 - 5z + 10)$. **[3 Marks]**

(ii) $\lim_{z\to -2i}\frac{(2z+3)(z-1)}{z^2-2z+4}$ **[4 Marks]**

6. a) Find the bilinear transformation that maps the points $z_1 = -i,\ z_2 = 0,\ z_3 = i$ into the points $w_1 = -1$, $w_2 = i,\ w_3 = 1$ respectively. Into what curve that $y-$ axis is transformed to this transformation? **[9 Marks]**

b) Find the modulus and the argument of this complex number $\frac{1-i}{1+i}$. **[3 Marks]**