



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
September Examination 2020_1

Course Code: MTH 307

Course Title: Numerical Analysis II

Credit Unit: 3

Time Allowed: 3 Hours

Instruction: Answer Question Number One and Any Other Four Questions

- 1a. Let $f(x)$ be a continuous function for $a \leq x \leq b$, then $C[a, b]$ the set of all continuous real valued function in the interval $[a, b]$, State the infinite or Chebyshev norm and its properties. **(5 marks)**
- b. Find the Cubic approximation to e^x by using Chebyshev polynomial. **(8 marks)**
- c. Generate the Chebyshev polynomial up to degree 5 in power of x **(5 marks)**
- d. Define $P_n(x)$ by RODRIGUE'S formula and state the three properties of $P_n(x)$. **(4 marks)**
- 2a. Convert the first five term of the Taylor Series Expansion for e^x into Chebyshev Polynomial. **(5 marks)**
- b. Construct the cubic Spline Interpolant to $f(x) = x$ with knots $-1, 0, 1$ subject to clamped boundary conditions. **(7 marks)**
- 3a. Consider the boundary Value problem $U'' + (1 + x^2)U + 1 = 0$, $U(\pm 1) = 0$. Determine the coefficient of the approximate solution $w(x) = a_1(1 - x^2) + a_2x^2(1 - x^2)$ by using the least square method. **(6 marks)**
- b. Use Taylor series approach to solve
- $$y'' + y = 0, \quad y(0) = 0, \quad y'(0) = 1 \quad \text{for } h = 0.05 \text{ at } x = 0.1 \quad \textbf{(6 marks)}$$

4a. Compute the min-max polynomial $q_1^*(x)$ to e^x on interval $[-1, 1]$. Hence, tabulate the error at $[-1, 1]$. **(4 marks)**

b. Estimate $\ln(2.7)$ from the following data

x_i	2	2.5	3
$\ln x_i$	0.693147	0.916291	1.098612
$1/x$	0.5	0.4	0.333333

(8 marks)

5a. Prove the orthogonality of Chebyshev polynomial with respect to weight function

$w(x) = (1 - x^2)^{-\frac{1}{2}}$ where

$$\int_{x_0=-1}^{x_1=1} T_n(x)T_m(x)w(x)dx = \begin{cases} 0, & m \neq n \\ \frac{\pi}{2}, & m = n \neq 0 \\ \pi, & m = n = 0 \end{cases} \quad \text{(8 marks)}$$

b. Find the fourth degree least square polynomial of $1/x$ over $[-1, 1]$ by means of Legendre Polynomial. **(4 marks)**

6a. Calculate a linear least square approximation to $f(x)$, if $f(x) = e^x$ on interval $[-1, 1]$.

(7 marks)

b. Evaluate $\int_0^{\frac{\pi}{3}} \sin x dx$ with $h = \pi/12$, correct to 5 decimal places using Trapezoidal rule.

(5 marks)