



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
September Examination, 2020_1

Course Code: MTH 315
Course Title: Analytic Dynamics
Credit Unit: 3
Time Allowed: 3 Hour
Instruction: Answer Question Number one and any Other Four Questions

1. (a) Define the following:
- (i) Total kinetic energy of a system of N particles **(2 marks)**
 - (ii) Centre of mass of a system of N particles **(2 marks)**
 - (iii) When does the total angular momentum is said to be conserved? **(3 marks)**
- (b) (i) Define the rest frame of a rigid body. **(2 marks)**
- (ii) Prove that if P and Q are two points fixed a rigid body and r is the vector from P to Q , then the velocities V_P and V_Q of P and Q relative to a frame \tilde{R} are related by $V_P = V_Q + \omega_r$ when ω_n the angular velocity of the body relative to \tilde{R} **(4 marks)**
- (c) (i) Discuss the motion of a body that glides frictionless on a uniformly rotating wire, r is its distance from the center of rotation. Given are the initial conditions:
 $r(t = 0) = r_0, \dot{r}(t = 0) = -r_0 \omega, \omega = \text{constant angular velocity of the wire}$ **(3 marks)**
- (ii) A parabolically curved wire rotates with constant angular velocity ω around the z axis. on this rotating wire, a bead of mass m moves frictionless in the earth's gravitational field ($g = -g e_z$). If the wire is just within the yz plane then it holds for the position of the mass $z = \alpha y^2$ ($\alpha > 0$)
- (a) Find the constraints **(2 marks)**
 - (b) How many degrees of freedom are left ? **(2 marks)**
 - (c) Use cylindrical coordinates (ρ, φ, z) to represent the lagrangian. **(2 marks)**
2. Consider a planar thread pendulum with the thread length l in the homogeneous gravitational field only small deflections of the pendulum are to be discussed.
- (a) Find the langrangian and the equation of motion, choose the initial conditions such that at time $t = 0$ the pendulum swings through its equilibrium position, how big is the frequency ω_0 of the oscillation ? **(4 marks)**
 - (b) Calculate the thread tension **(4 marks)**
 - (c) Show that $F = x^2 y z_i - x y z_k^2$ is non-conservative **(4 marks)**

3. (a) Define the following terms:
- (i) Amplitude of a motion (2 marks)
 - (ii) Period of a motion (2 marks)
 - (iii) Simple Harmonic motion (2 marks)
- (b) A particle is moving with simple harmonic motion of period 4π about a centre 0, it passes through a point distance $4m$ from 0. Find the time which elapses before it next passes through this point. (6 marks)
4. (a) One end of an elastic string of length $24cm$ is fixed ended and to the other suspended end, a mass of $5kg$ is attached, which when in equilibrium stretches the string $4cm$. The mass is pulled down at a distance of $3cm$ below its equilibrium position and then released. Find the period of oscillation and the maximum kinetic energy of the mass. (6 marks)
- (b) Three forces of magnitude $15Q, 10Q, 5Q$ act on a particle in directions which make 120° with one another. Find their resultant. (6 marks)
5. (a) Due to a force field, a particle of mass 5 units moves along a space curve whose position vector is given as a function of time t by $r = (2t^2 + t)i + (3t^4 - t^2 + 8)j - 12t^2k$
Find:
- (i) the velocity, (2 marks)
 - (ii) the momentum (2 marks)
 - (iii) the acceleration and (2 marks)
 - (iv) the force field at any time t . (2 marks)
- (b) A particle of mass m moves in the xy plane so that its position vector is $r = a\cos wt i + b\sin wt j$
Where a, b and w are positive constant and $a > b$.
- (i). Show that the particle moves in an ellipse. (2 marks)
 - (ii). Show that the force acting on the particle is always directed toward the origin. (2 marks)
6. Show that for the functions $f = f(\mathbf{q}, \mathbf{p}, t); \quad g = g(\mathbf{q}, \mathbf{p}, t) \quad h = h(\mathbf{q}, \mathbf{p}, t)$
the following relations are valid:
- i. $\frac{\partial}{\partial t} \{f, g\} = \left\{ \frac{\partial f}{\partial t}, g \right\} + \left\{ f, \frac{\partial g}{\partial t} \right\}$ (4 marks)
 - ii. $\frac{d}{dt} \{f, g\} = \left\{ \frac{df}{dt}, g \right\} + \left\{ f, \frac{dg}{dt} \right\}$ (4 marks)
 - iii. $\{f, g, h\} = g\{f, h\} + \{f, g\}h$ (4 marks)