NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.

FACULTY OF SCIENCES

2020_1 EXAMINATIONS

Course Code: MTH402
Course Title: General Topology II
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Attempt Number One (1) and Any Other Four (4) Questions

1. (a) Define a topological space. **(3 marks)**
   (b) Show that the intersection $\tau = \bigcap_\alpha \tau_\alpha$ of topologies $\{\tau_{\alpha\in\Delta}$ on X is itself a topology in X$\}$, where Δ is some indexing set. **(14 marks)**
   (c) Let X and Y be two topological spaces. Let **B** be the collection of all sets of the form U x V, where U is an open subset of X and V is an open subset of Y. i.e.,
   B := {U x V : U is open in X and V is open in Y}.
   Show that **B** is basis for topology on X x Y. **(3 marks)**
   (d) Let Y be a subspace of X. If U is open in Y and Y is open in X, show that U is open in X. **(2 marks)**
2. (a) Define the following terms:
   (i) basis for a topology on a set X. **(3 marks)**
   (ii) topology generated by a basis. **(3 marks)**
   (b) Let **B** and $\mathbf{B^0}$ be bases for the topologies τ and τ^0 respectively on X. Show that the following are equivalent:
   i. τ^0 is finer than τ. **(3 marks)**
   ii. For each $x \in X$ and each element B ∈ **B** containing x, there exists a basis element $B^0 \in \mathbf{B^0}$ such that $x \in B^0 \subset$ **B.** **(3 marks)**

3. (a) Let d be a metric on the set X. Show that the collection of all r - balls B_d (x, r), for x ∈X and r > 0 is a basis for a topology on X, called the metric topology induced by d. **(6 marks)**

   (b) Prove that the collection
   S = $\{\pi_1^{-1}(U): U \text{ is open in } X\} \cup \{\pi_2^{-1}(V): V \text{ is open in } Y\}$
   is a subbasis for the product on X x Y. **(6 marks)**

4. (a) Let Y be a subspace of X. Show that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y. **(6 marks)**
   (b) Let A be a subset of the topological space X. Prove that:

(i) The $x \in \bar{A}$ if and only if every open set U containing x intersects A.

(ii) Supposing the topology of X is given by a basis, then $x \in \bar{A}$ if and only if every basis element B containing x intersects A. **(6 marks)**

5. (a) Let A be a subset of the topological space X. Let A^0 be the set of all limit points of A. Show that $\bar{A} = A \cup A^0$. **(6 marks)**

(b) State whether each of the following is a Hausdorff space or not:

(i) Every metric topology. $\left(\mathbf{1\frac{1}{2}\ marks}\right)$

(ii) Every discrete space. $\left(\mathbf{1\frac{1}{2}\ marks}\right)$

(iii) The real line R with the finite complement topology. $\left(\mathbf{1\frac{1}{2}\ marks}\right)$

(iv) R with the usual topology. $\left(\mathbf{1\frac{1}{2}\ marks}\right)$

6. (a) Show that if X is a Hausdorff space, then for all $x \in X$, the singleton set $\{x\}$is closed. **(7 marks)**

(b) Let X be a Hausdorff space, then a sequence of points of X converges to at most one point of X. (i.e., if a sequence $\{x_n\}$ in X, a Hausdorff space, converges, the limit is unique. **(5 marks)**