NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

2020_2 EXAMINATION

Course Code: MTH 303
Course Title: MTH 303 Vector and Tensor Analysis
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Question Number One and Any other Four Questions.

1. a) Find a unit vector parallel to the resultant of vectors $r_1 = 2i + 4j - 5k$ and $r_2 = i + 2j + 3k$ [4 Marks]

b) Calculate the *curl* of the vector $\vec{f} = xyzi + 3x^2yj + (xz^2 - y^2z)k$ [4 Marks]

c) If $A = i - 2j - 3k$, $B = 2i + j - k$ and $C = i + 3j - 2k$, find $A \bullet (B \times C)$. [5 Marks]

d) If $f = f(x^1, x^2, x^3, \ldots x^n)$, then show that $df = \dfrac{\partial f}{\partial x^i} dx^i$. [4 Marks]

e) Using Stoke's theorem or otherwise, evaluate $\int_C \left[(2x - y)dx - yz^2dy - y^2zdz\right]$ where C is the circle $x^2 + y^2 = 1$, corresponding to the surface of sphere of unit radius. [5 Marks]

2a) Determine λ and μ by using vectors, such that the points A, B and C are given as $(-1, 3, 2)$, $(-4, 2, -2)$ and $(5, \lambda, \mu)$ respectively lie on a straight line. [6 Marks]

b) Evaluate $\iint_{\Re} \sqrt{x^2 + y^2}\, dxdy$, where $\Re$ is the region bounded by $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$ [6 Marks]

3a) If $A = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int_C A \bullet dr$ from (0,0,0) to (1,1,1) along $x = t$, $y = t^2$ and $z = t^3$. [8 Marks]

b) If the coordinates of P be $(3, 4, 12)$, then find $\overrightarrow{OP}$, its magnitude and direction cosines. [4 Marks]

4a) Apply the divergence theorem to compute $\iint_S u \cdot n\, ds$ where S is the surface of the cylinder $x^2 + y^2 = a^2$ bounded by the planes $z = 0$, $z = b$ and where $u = xi - yj + zk$. [6 Marks]

b) Let $F = \dfrac{-yi + xj}{x^2 + y^2}$. Calculate $\nabla \times F$ [6 Marks]

5a) Evaluate the integral $\iiint_{\Re}(x^2+y^2+z^2)dxdydz$ where $\Re$ is the region bounded by $x+y+z=a\ (a>0)$ at $x=0$, $y=0$ and $z=0$. [8 Marks]

b) Write the covariant derivative with respect to x^q of this tensor A^{jk} [4Marks]

6. a) Evaluate $\int_{(0,1)}^{(1,2)}(x^2-y)dx+(y^2+x)$ along

(i) a straight line from $(0,1)$ to $(1,2)$ [3 Marks]

(ii) a straight lines from $(0,1)$ to $(1,1)$ and then from $(1,1)$ to $(1,2)$ and [3 Marks]

(iii) the parabola $x=t$ and $y=t^2+1$. [3 Marks]

b) Express in matrix notation the transformation equations for a covariant vector $\overline{A}_q=\frac{\partial x^q}{\partial \overline{x}^p}A_q$ of rank two, assuming $N=3$. [3 Marks]