NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2020_2 EXAMINATION

Course Code:	**MTH304**
Course Title:	**Complex Analysis I**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One (1) and Any Other 4 Questions**

1. (a) Given that $z_1 = (a_1, b_1)$, $z_2 = (a_2, b_2)$, $z_3 = (a_3, b_3)$ then prove the distributive law: $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$ **(6 marks)**

   (b) Given $z_1 = -12 + 5i$ and $z_2 = 2 - 3i$, show that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ **(6 marks)**

   (c) Given that $z_1 = 2 + i$ and $z_2 = 3 - 2i$, then evaluate $|z_1 z_2|$ **(4 marks)**

   (d) Find the square root of the complex number $3 + 2i$ **(6 marks)**

2. (a) Let $w = 3iz + z^2$ and $z = x + iy$. Find $|w|^2$ in terms of x and y. **(6 marks)**

   (b) Find the real and imaginary parts of the following

   (i) $w = 2iz^2$ **(3 marks)**

   (ii) $w = (2 - i)\bar{z}$ **(3 marks)**

3. Write each of the following equations in terms of conjugate coordinates.

   (i) $x^2 + y^2 = 4$ **(4 marks)**

   (ii) $x - 3y = 23$ **(4 marks)**

   (iii) $x^2 + 4y^2 = 9$ **(4 marks)**

4. (a) Evaluate each of the following using theorems on limits

   (i) $\lim\limits_{z \to 1+i} (z^2 - 4z + 4)$ **(3 marks)**

   (ii) $\lim_{z \to -3i} \frac{(z+3)(z-1)}{z^2 - 2z + 1}$ **(3 marks)**

   (b) Prove that if $|a| < 1$

$$1 + a\cos\theta + a^2 \cos 2\theta + a^3 \cos 3\theta + \cdots = \frac{1 - a\cos\theta}{1 - 2a\cos\theta + a^2}, \text{ and}$$

$$a \sin\theta + a^2\, sin\, 2\theta + a^3\, sin\, 3\theta + \cdots = \frac{a\, sin\, \theta}{1-2a\, cos\, \theta + a^2}.$$ **(6 marks)**

5. Prove the identities

(a) $\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4} sin\, 3\theta$ **(4 marks)**

(b) $\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2} cos\, 2\theta + \frac{3}{8}$ **(5 marks)**

(c) $\sin^2\theta + \cos^2\theta = 1$ (**3 marks**)

6. Verify Green's theorem in the plane for

$\oint_C (2xy - x^2)dx + (x + y^2)dy$ **(12 marks)**

where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$.