



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS

Course Code: MTH 305

Course Title: Complex Analysis II

Credit Unit: 3

Time Allowed: 3 Hours

Instruction: Answer Question Number One and Any other Four Questions.

- 1a) Express $z = -\sqrt{6} - \sqrt{2}i$ in polar form. [4 Marks]
- b) If $f(z) = z^2$, prove that $\lim_{z \rightarrow z_0} z^2 = z_0^2$. [4 Marks]
- c) Find the analytic function $f(z) = u + iv$, given that $u = e^{-x}(x \sin y - y \cos y)$. [4 Marks]
- d) If $A(x, y) = 2xy - i x^2 y^3$, find $\text{div } A$. [4 Marks]
- e) Find the residue of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane. [6 Marks]
2. a) Given the complex function $f(z) = \frac{1}{(z^2 + 4)}$. Find the first four terms of the Taylor series expansion $f(z)$ about $z = -i$. [7 Marks]
- b) Show that the function $e^x(\cos y + i \sin y)$ is an analytic function, find its derivative. [5 Marks]
- 3a) Write all possible Laurent series for the function $f(z) = \frac{1}{z(z+2)^3}$ about the pole $z = -2$ [7 Marks]
- b) Find $\frac{df}{dz}$ of this function: $f(z) = 4x + y + i(-x + 4y)$ along
- (i) imaginary axis [3 Marks]
- (ii) a line when $y = x$ [2 Marks]

4. a) Find the image of the circle $|z - 1| = 1$ in the complex plane under the mapping $w = \frac{1}{z}$. [7 Marks]

b) Evaluate $\int_{3i}^{2+4i} (2y + x^2) dx + (3x - y) dy$ along the parabola $x = 2t$ and $y = t^2 + 3$. [5 Marks]

5a) Evaluate the following integrand using Cauchy integral formula $\int_c \frac{4-3z}{z(z-1)(z-2)} dz$ where c is the circle $|z| = \frac{3}{2}$. [8 Marks]

b) Find the region of convergence of the series $\sum_{n=1}^{\infty} n! z^n$. [4 Marks]

6a) Find the smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$ [5 Marks]

b) If $z_1 = 2 + i$ and $z_2 = 3 - 2i$, show that $\left|\frac{2z_2 + z_1 - 5 - i}{2z_1 - z_2 + 3 - i}\right|^2 = 1$. [7 Marks]