

**NATIONAL OPEN UNIVERSITY OF NIGERIA**

**14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS**

**SCHOOL OF SCIENCE AND TECHNOLOGY**

**JUNE/JULY EXAMINATION**

**COURSE CODE: MTH301**

**COURSE TITLE: METRIC SPACES (3 units)**

**TIME ALLOWED:3 HOURS**

**INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE(5) QUESTIONS BEAR FULL MARKS**

1(a) What is a metric space? Give one example of a metric space. -4marks

1(b) What is a topological space? Give an example of a topological space.

-4marks

1(c) Define the length or norm of a vector x ε R3-6marks

2(a) Let X ε Rn.Show that the set B(X, ε ) is open.-6marks

2(b) Let X be a complete metric space and {On} is countable collection of dense open subset of X. Show that Onis not empty. -8marks

3 Let f and g be real-valued functions with Domain f = Range(g) = D Rn.

Let x0 be a point of accumulation on D. If the = and

 **=** n.

1. If for α, β ε **R**, show that (αf + βg)(x) =  -4marks
2. Show also that  =  -4marks
3. If g(x)  0; for X ε D and 

Show that  = =  -4marks

4 Let (X,d) and (Y,d) be metric spaces and f a mapping of X into Y. Let τ1 and τ2 be the topologies determined by d and d1 respectively. Then f(X, τ)(y, τ) is continuous if and only if ; that is if x1, x2, . . . , xn, . . . , is a sequence of points in (X, d) converging to x, show that the sequence of points f(x1), f(x2), . . . , f(xn), . . . in (Y, d) converges to x. -14marks

5(a) Prove that for any y, z ε , max(y,z) = ½[y+z+ |y-z|], min(y,z) = ½[y+z- |y-z|].

-6marks

5(b) Let f,g: be continuous at aε. Show that h,k:  defined through

h(x) = ,k(x) =  are continuous at a.

-8marks

6 Let M = { A, d} be a metric space. Given any four points x, y, z, t ε A. Prove that

d(x, z) + d(y, t) ≥ |d(x, y)- d(z, t)| -14marks

7 Show that the mapping f **R****R+**defined by f(x) = e**x**is a homeomorphism from **R**ontoR+(A homeomorphism from one topological space to another is a bijective function) -14marks