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**NATIONAL OPEN UNIVERSITY OF NIGERIA**

**14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS**

**SCHOOL OF SCIENCE AND TECHNOLOGY**

**JUNE/JULY EXAMINATION**

**COURSE CODE: MTH402**

**COURSE TITLE: GENERAL TOPOLOGY II (3 units)**

**TIME ALLOWED: 3 HOURS**

**INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE (5) QUESTIONS BEAR FULL MARKS**

1(a) Prove that the intersection =   of topologies { τα} αε∆ on X its itself a topology in X ( where ∆ is some indexing set) -7marks

1(b) Let X be a set, and let B be a basis for a topology on τ on X. Show τ equals the collection of all unions of elements of B. -7marks

2(a) Let B and B’ be bases for the topology τ and τ’ respectively on X. Show that the following are equivalent.

1. τ’ is finer than τ
2. For each x ε X and each basis element в ε B such that x ε B, we know that в ε τ by definition and that τ τ’, by condition (i) therefore в ε τ’ -7marks

2(b) Let d be a metric on the set X. show that the collection of all ε – balls Bd(x, ε) for x ε X and ε > 0 is a basis for a topology on X, called the metric topology induced by d. -7marks

3(a) State the properties under which d is a metric on X, given a function d: X x X → R, for all x, y, z ε X. -6marks

3(b) Let X and Y be two topological spaces. Let B be the collection of all sets of the form U x V, where U is an open subset of X and V is an open subset of Y i.e. B = { U x V: U is open in X and V is open in Y}. Show that B is the basis for a for a topology on X x Y. -8marks

4(a) Let Y be a subspace of X. If U is open in Y and Y is open in X.

Show that U is open in X. -7marks

4(b) Show that the mapping f: R → R+ defined by f(x) = ex is a homeomorphism from R to R+ . ( Recall that a homeomorpism from one topological space to another is a bijective function f such that f and f -1 are both continuous) -7marks

5(a) Give an example to show that every discrete space is Hausdorff. -2½marks

5(b) What does it mean to say that a topological space is Hausdorff? -2½marks

5(c) Let X be the Hausdorff space, then for all x ε X, show that the singleton

set {x} is closed. -9marks

6(a) Let X and Y be topological spaces. When is a function f: X → Y said to be

continuous? -7marks

6(b) Let X be the subspace of R given by X = [0,1] U [2,4], Define f : X → R by

f(x) = . Prove that f is continuous. -7marks

7(a) Let R be endowed with standard topology. Show that for all x ε R, w = { ( x – ε, x + ε) ε > 0} is a neighbourhood basis of x. -7marks

7(b) Let X be a topological space and let x ε X. Suppose X is first countable, show that there exist a countable basis of x, say W = { Wn n ≥ 1 } such that Wn+1 Wn -7marks