

**NATIONAL OPEN UNIVERSITY OF NIGERIA**

**14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS**

**SCHOOL OF SCIENCE AND TECHNOLOGY**

**JUNE/JULY EXAMINATION**

**COURSE CODE: MTH 411**

**COURSE TITLE: MEASURE AND INTEGRATION (3units)**

**TIME ALLOWED:3 HOURS**

**INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE (5) QUESTIONS BEAR FULL MARKS**

1(a) If E1, E2, E3 . . . , En are pairwise disjoint measurable subsets of **R’,** show that μ\*( X ) =  for every subset X of **R’** 7marks

1(b) Show that the function ,is translation invariant i.e for every **R’** and E **R’** we have μ\*{ Tr(E)} = μ\*(E); where the translation function Tr is defined by Tr(x) = x + r for every x ε **R’.** 7marks

2(a) Show that a subset E of **R’** is Lebesque measurable if and only if every subset X of **R’** we have  =  + 9marks

2(b) Prove that if a subset E of **R’** is measurable, then so is its complement 5marks

3(a) Explain carefully what is meant by the LebesqueOuter Measure μ\*(E) of a subset E of the real line **R**  5marks

3(b) Prove that for any two subsets A and B of **R**, if A B, then μ\*(A) ≤ μ\* (B) 9marks

4(a) Prove that if two subsets A and B of the real line are measurable, then so is *A*

7mark

4(b) Prove that for every countable family

 of subsets of **R,** we have μ\* ≤ 7marks

5(a) Find the length of the set 7marks

5(b) Prove that if E is any countable set of real numbers, the = 0 7marks

6(a) Define a set with measure Zero. 7marks

6(b) Suppose f = g almost everywhere. Show that

  =  7marks

7(a) Let E1, E2, E3 . . . , En be disjoint measurable subset of E with μ(E)<, then

Every linear combination S = with real coefficient a1, a2, a3, . . . am is

measurable simple function and IE(s) = .7marks

7(b) Prove that a monotonic increasing sequence of measurable sets in **R**’satisfies the relation  = 7marks