

**NATIONAL OPEN UNIVERSITY OF NIGERIA**

**14/16 AHMADU BELLO WAY, VICTORIA ISLAND, LAGOS**

**SCHOOL OF SCIENCE AND TECHNOLOGY**

**JUNE/JULY EXAMINATION**

**COURSE CODE: MTH 412**

**COURSE TITLE: Normed Linear Spaces**

**TIME ALLOWED:3 HOURS**

**INSTRUCTION: COMPLETE ANSWERS TO ANY FIVE (5) QUESTIONS BEAR FULL MARKS**

**INSTRUCTION:**

1(a) Define Linear Maps. 5marks

1(b) Let X = *l*2. For each = ( x1, x2, x3, . . . xk, . . . .) in *l*2. Show that if T = T (x1, x2, x3, . . .xk, . . . .) = ( 0, x1, x2⁄2, x3⁄3, . . . xk⁄k,. . ..), then T is a linear

map on *l*2.

9marks

2(a) Let ( X, ρ) be a metric space. Define Cauchy sequence. 5marks

2(b) Let ( X, ρ ) be a complete metric space, and let EX. Show that ( E, ρE)is complete if and only if it is closed. (Where ρEis the subspace metric induced

by ρ). 9marks

3(a) Define the convergence of a sequence { xn} of elements of X to a point x ε X?

5marks

3(b) Let X = [-3, 3] with 2 = show that X is not complete.

9marks

4(a) The surface of a unit sphere centered around the origin of a linear space with the $ \ell^p$-norm is the locus of points $ \{ (x_1,x_2,\cdots\}$. Show that

$\displaystyle \left(\sum^\infty_{k=1}\vert x_k\vert^p\right)^{\frac{1}{p}}=1~.
$ 7marks

4(b) Let X = **R**2. For each vector = ( x1, x2) ε X. Define ||.||2 : X →R by ||x||2 = ()½.Show that ||.|| is a norm on X. 7marks

5(a) Let X = c[0,1] = Y, where c[0,1] is endowed with the supnorm.

Let D = {f ε c’[0,1] : f’ε c[0,1]} where the prime denotes differentiation.

Let T : c[0,1] → c[0,1] be a map with domain D defined by Tf = f’ (i.e. differentiation operator). Show that:

1. T is linear 2½marks
2. T is closed. 2½marks

5(b) Show that an inner product space E becomes a normed linear space when equipped with the norm = for all x ε E. 9marks

6(a) Let X be a linear space over a scalar field K = (**R** or **C**)**.** When is a function ||.|| : X →R said to be a norm (in X)? 5marks

6(b) Show that the real line **R** becomes a normed linear space if you set ||x|| =|x| for every number x ε **R.** 9marks

7(a) What is aConvex set? If xεRnand if r >0; show that the ball

B(x\*, r) ={ yεRn: ||y - x\*||< r}centred at x\* of radius r is a convex set

7marks

7(b) Let x, and v be vectors in **R**n, show that the line L through x in the direction of v given by L = { x+αv : αε**R}** is a convex set**.** 7marks