NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
AprillMay, 2019 Examinations

| Course Code: | MTH303 |
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| Course Title: | Vector and Tensor |
| Credit Unit: | $\mathbf{3}$ |
| Time Allowed: | 3 Hours |
| Total: | 70 Marks |
| Instruction: | Answer Question One and Any Other 4 Questions |

1. (a) Let $A=A_{1} i+A_{2} j+A_{3} k$ and $B=B_{1} i+B_{2} j+B_{3} k$, then prove that
(i) $\nabla(A B)=A \nabla B+B \nabla A$
(ii) $\nabla \cdot(A+B)=\nabla \cdot A+\nabla \cdot B$ (6 marks)
(iii) $\nabla \times(\mathrm{A}+\mathrm{B})=\nabla \times \mathrm{A}+\nabla \times \mathrm{B}$ (6 marks)
(b) If $A=x z^{3} i-2 x^{2} y z j+2 y z^{4} k$, find $\nabla \times A$ at point $(1,-1,1)$
(6 marks)
2. (a) Prove that, for every field $A ; \nabla \cdot(\nabla \times A)=0$ (6 marks)
(b) If $\phi(x, y, z)=x y^{2}$ and $A=x z i-x y^{2} j+y z^{2} k$, find $\frac{\partial^{3}}{\partial x^{2} \partial z}(\phi A)$ at point $(2,-1,1) \quad$ ( 6 marks)
3. (a) If $A=3 i-j+2 k, B=2 i+j-k$ and $C=i-2 j+2 k$, find $(A \times B) \times C$
(6 marks)
(b) A particle moves along the curve $A=\left(t^{3}-4 t\right) i+\left(t^{2}+4 t\right) j+\left(8 t^{2}-3 t^{3}\right) k$, where $t$ is the time. Find the magnitude of the tangential components of its acceleration at $t=2$. (6 marks)
4. (a) If $A(t)=\left(t-t^{2}\right) i+2 t^{3} j-3 k$, find $\int_{1}^{2} A(t) d t$ (6 marks)
(b) Let $A=3 x i+x^{2} j+(x+2) k$ and $B=2 x i-3 x j+(x-2) k$, evaluate $\int_{0}^{2}(A \times B) d x$ ( $\mathbf{6}$ marks)
5. (a) If $\phi=3 x^{2} y-y^{3} z^{2}$; find $\operatorname{grad} \phi$ at point $(1,-2,-1)$
(b) Find the divergence and curl of $A=(x y z) i+\left(2 x^{2} y\right) j+\left(x z^{2}-y^{2} z\right) k$ at $(2,-1,1)$ (7 marks)

6 (a) If $A_{r}^{p q}$ and $B_{r}^{p q}$ are tensor. Prove that their sum and difference are tensor
(6 marks)
(b) If $A=2 i-j+k, B=i+3-2 k, C=-2 i+j-3 k$ and $D=3 i+2 j+5 k$.

Find scalar $\mathrm{p}, \mathrm{q}$ and r such that $D=p A+q B+r C$
(6 marks)

