NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
Department of Mathematics
2021 Examinations

Course Code: MTH307
Course Title: Numerical Analysis II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

1. (a) Given the general second order Partial Differential Equation

$$L(u) = Au_{xx} + Bu_{xy} + Cu_{yy} - H(x, y, u, u_x, u_y) = 0$$

Classify the following equation into Parabolic, Elliptic and Hyperbolic.

i. $u_t = u_{xx}$

ii. $u_{tt} = u_{xx}$

iii. $u_{xx} + u_{yy} = 0$ **(5 marks)**

(b) When is Partial Differential Equation said to be Parabolic, Elliptic and Hyperbolic.

(5 marks)

(c) Solve the Laplace equation $u_{xx} + u_{yy} = 0,$ subject to the boundary conditions $u(x,0) = 1, u(0,y) = 0, u(1,y) = 0, u(x,1) = 1;\ 0 \le x \le 1,\ 0 \le y \le 1.$ **(12 marks)**

2. (a) Express the function $x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. **(5 marks)**

(b) Find the fourth degree least square polynomials to $|x|$ over $[-1,\ 1]$ by means of Legendre polynomials. **(7 marks)**

3. (a) Show that $T_n(x)$ satisfies the differential equation $(1-x^2)y'' - xy' + n^2 y = 0$

(5 marks)

(b) Convert the first 5 terms of the Taylor series expansion for e^x into Chebyshev

polynomials. **(7 marks)**

4. (a) Determine the parameters in the formula

$p(x) = a_o(x-a)^3 + a_1(x-a)^2 + a_2(x-a) + a_3$ such that

$p(a) = f(a),\ \ p'(a) = f'(a),\ \ p(b) = f(b),\ \ p'(b) = f'(b).$ **(5 marks)**

(b) Obtain the unique polynomial $p(x)$ of degree 5 or less approximating the function

$f(x)$, where $f(x_0) = 1,\ f'(x_0) = 2.$ Also, find $p\left(\frac{(x_0 + x_1)}{2}\right).$ **(7 marks)**

5. (a) Distinguish between Trapezoidal and Simpson's rule. **(5 marks)**

(b) Evaluate $\int_1^3 \frac{1}{x+1} dx$ using the Simpson's one-third rule with $h = \frac{1}{4}$, working with four floating point arithmetic. **(7 marks)**

6. (a) State the properties of a cubic spline interpolation. **(5 marks)**

(b) Obtain the cubic spline from data below and compute $y(1.5)$ and $y'(1)$.

X	1	2	3
Y	–8	–1	18

(**7 marks**)