NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_1 Examination

Course Code: **MTH 309**
Course Title: **Optimization Theory**
Credit Unit: **3**
Time Allowed: **3 Hours**
Total: **70 Marks**
Instruction: **Answer Question Number one and any Other Four (4) Questions**

1. A farmer requires 10, 12, and 12 units of chemicals A, B, and C respectively for his garden, a liquid product contains 5, 2, and 1 unit of A, B, and C respectively per jar. A dry product contains 1, 2, and 4 units of A, B, and C per carton. If the liquid product sells for $3 per jar and the dry product sells for $2 per carton. How many of each should be purchased in order to minimize the cost and meet the requirements?
   (a). Formulate the problem as LPP **(5 marks)**
   (b). Solve the LPP by graphical method **(5 marks)**
   (c). Write the dual of the LPP **(5 marks)**
   (d). Solve the dual by Simplex method and obtain the primal solution **(7 marks)**

2. (a). i. Outline the phases of Operations Research/Optimization **(1 mark)**
   ii. In your own words explains the following terms:
   - Basic Feasible solutions BFS **(1 marks)**
   - Optimum solution **(1 marks)**
   - A generate and degenerate BFS **(1 marks)**

   (b). Given the system

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

   Classify as feasible and infeasible all the basic solutions of the given system of equations. **(4 marks)**

   (c). Given the Objective function $Maximize\, z = x_1 + 4x_2 + 7x_3$. Generate the simplex tableau associated with the basis $B = \begin{pmatrix} 1 & 3 \\ 2 & -2 \end{pmatrix}, b = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ **(4 marks)**

3. Using two – phase method solve the following LPP

$$Maximize\, z = -4x_1 - 3x_2 - 9x_3$$

Subject to $2x_1 + 4x_2 + 6x_3 \geq 15$

$6x_1 + x_2 + 6x_3 \geq 12$

$x_1, x_2, x_3 \geq 0$ **(12 marks)**

4. Suppose an automobile company has three plants, A, B, and C and two distributions centers X and Y. The capacities of the three plants during the next quarter aee 1000, 1500, and 1200 cars respectively. The quarterly demand at the two distribution centers are 2300 and 1400 cars respectively. The transportation cost per car on the routes rounded to the nearest dollar is given below:

	X	**Y**
A	\$80	\$215
B	\$100	\$108
C	\$102	\$68

(a). Construct the LP model of the problem in terms of $Ax = b$ **(4 marks)**

(b). Solve the transportation problem starting with North-West corner method **(8 marks)**

5. Suppose one of the solutions to the IP relaxation of the following IP

$Maximize\ z = 13x_1 + 8x_2$

Subject to $x_1 + 2x_2 \leq 10$

$5x_1 + 2x_2 \leq 20$

$x_1, x_2 \in \mathbb{Z}_+$

is $x_1^0 = 2.5$. Create two new subproblems $LP^1, and\ LP^2$ from $x_1^0 = 2.5$, and solve to optimality the subproblem with all slack starting solution. **(12 marks)**

6. Generate Gomory fractional cut from the tableau below, using the x_2 −row as a source row and solve the problem to optimality. **(12 marks)**

Basis	x_1	x_2	x_3	x_4	RHS
x_2	0	1	$^7/_{22}$	$^1/_{22}$	$^7/_2$
x_1	1	0	$^{-1}/_{22}$	$^3/_{22}$	$^9/_2$
$z_j - c_j$	0	0	$^{28}/_{11}$	$^7/_{22}$	63