NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES

April /May Examination 2019

Course Code:	**MTH309**
Course Title:	**Optimization Theory**
Credit Unit:	**3**
Time Allowed:	**3 HOURS**
Total:	**70 Marks**
Instruction:	**ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) Define Optimization Problem in R^n or simply an Optimization Problem **(2 marks)**

(b) Distinguish between; (i) Unconstrained Optimization Problem and **(2.5 marks)**

(ii) Constrained Optimization Problem **(2.5 marks)**

(c) Differentiate between; (i) A solution to the problem $max\{f(x)|x \in D\}$ **(2 marks)**

(ii) A solution to the problem $min\{f(x)|x \in D\}$ **(2 marks)**

(d). Use the Simplex method to solve the following linear programming problem;

Maximize $Z = 2x_1 - x_2 + 2x_3$

Subject to; $2x_1 + x_2 \leq 10$

$x_1 + 2x_2 - 2x_3 \leq 20$

$x_2 + 2x_3 \leq 5$

$x_1,\ x_2,\ x_3 \geq 0$ **(11 marks)**

2. (a) Define the dual Problem, if we let the primal problem be as stated below; **(5 marks)**

Maximize $Z = c_1x_1 + \cdots, +c_nx_n$

Subject to; $a_{11}x_1 + a_{12}x_2 + \cdots, +a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots, +a_{2n}x_n \leq b_2$

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$a_{m1}x_1 + a_{m2}x_2 + \cdots, +a_{mn}x_n \leq b_m$

$x_1,\ x_2,\ \ldots,\ x_n \geq 0$

(b) Write the dual of the following primal LP problem

Maximize $Z = x_1 + 2x_2 + x_3$

Subject to; $2x_1 + x_2 - x_3 \leq 2$

$-2x_1 + x_2 - 5x_3 \geq -6$

$4x_1 + x_2 + x_3 \leq 6$

$x_1,\ x_2,\ x_3 \geq 0$ **(7 marks)**

3. Solve the following linear programming problem

Minimize $W = 2x_1 + 10x_2 + 8x_3$

Subject to; $x_1 + x_2 + x_3 \geq 6$

$x_2 + 2x_3 \geq 8$

$-x_1 + 2x_2 + 2x_3 \geq 4$

$x_1,\ x_2,\ x_3 \geq 0$ **(12 marks)**

4. (a) Use penalty method to solve the following problem;

Maximize $Z = 3x_1 + 2x_2$

Subject to; $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1,\ x_2 \geq 0$ **(6 marks)**

(b) Use two-phase simplex method to solve;

Maximize $Z = 5x_1 + 3x_2$

Subject to; $2x_1 + x_2 \leq 1$

$x_1 + 4x_2 \geq 6$

$x_1,\ x_2 \geq 0$ **(6 marks)**

5. Find the dual of the following Linear programming problem;

Maximize $Z = 3x_1 - x_2 + x_3$

Subject to; $4x_1 - x_2 \leq 8$

$8x_1 + x_2 + 3x_3 \geq 12$

$5x_1 - 6x_3 \leq 13$

$x_1,\ x_2,\ x_3 \geq 0$ **(12 marks)**

6. Solve the following minimization problem;

Minimize $W = 3x_1 + 2x_2$

Subject to; $2x_1 + x_2 \geq 6$

$x_1 + x_2 \geq 4$

$x_1,\ x_2 \geq 0$ **(12 marks)**