NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.
FACULTY OF SCIENCE AND TECHNOLOGY

April/May Examination 2019

COURSE CODE: **MTH381**

COURSE TITLE: **MATHEMATICAL METHODS III**

CREDIT UNIT: **3**
TIME: **3 HOURS**

Total: **70 Marks**

INSTRUCTION: **Attempt question One (1), and any other four questions.**

Question 1

(a) If $f(x,y) = x^2 - 2xy + y^2$

Find (i) $f(1,-1)$ (ii) $f(2,1)$ [2 marks each]

(b) If $u = x + y + z$, $v = x^3 + y^3 + z^3$ and $w = xyz$

find the Jacobian $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$ [6 marks]

(c) Determine the Fourier series of the function defined by

$$f(x) = 2x \qquad 0 < x < 2\pi$$

$$f(x + 2\pi) = f(x)$$ [6 marks]

(d) Express the following in polar from stating the modulus of the vector and argument (the principal) of the vector value:

(i) $1 + i$ (ii) $-5 + 5i$ [3 marks each]

Question 2

(a) Determine whether the following pair of functions are linearly dependent as the case may be

(i) $u(x) = x^2$, and $v(x) = 3x^2$ [3 marks]
(ii) $u(x) = \cos 2x$, and $v(x) = \sin 2x$ [3 marks]

(b) State the Residue theorem. [2 marks]

(c) Evaluate $\oint_C \frac{e^z}{(z+1)^2}dz$ where C is the circle $|z-1|=3$ [4 marks]

Question 3

(a) Determine the poles of the function

$$\frac{z^3}{(z-1)^3(z+3)}$$

And the residue at each pole. [3 marks]

(b) Verify divergence theorem for the vector field
$F = zi + y^2 j + xk$
over the region bounded by the planes
$x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. [9 marks]

Question 4

(a) If $f(x, y) = \frac{3x+2y}{4-2xy}$

Find (i) $f(0,1)$ (ii) $f(1,3)$ [2 marks each]

(b) Using Laplace transformation, solve the initial value problem:

$y'' - 3y' - 2y = 4t;$ $y(0) = 1$ and $y'(0) = -1$ [8 marks]

Question 5

(a) Evaluate the double integral

(i) $\int_{y=1}^{y=2}\int_{x=0}^{x=3}(x^2 + y)dxdy$ (ii) $\int_1^2\int_1^3 x^2 ydxdy$ [6 marks each]

Question 6

(a) If $A = (2x^2 + 5y)i - 10yzj + 5xz^2k$ evaluate $\int_c A.dr$ from (0,0,0) to (1,1,1) along the following parts C: $x = t$, $y = t^2$, and $z = t^3$

The straight lines from (0,0,0) to (1,0,0) then to (1,1,0) and then to (1,1,1).

The straight line joining (0,0,0) and (1,1,1).

(b) Given that $z_1 = 3 - 4i$ & $z_2 = -6 + i$; find (i) $z_1 z_2$ (ii) z_1/z_2 [3 marks each]

End of Examination questions.