NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
November 2018 Examinations

Course Code:	**MTH381**
Course Title:	**Mathematical Methods III**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One (1) and Any Other 4 Questions**

1. (a) (i) By using Wronskian determine the functions $u(x) = 2\sinh x$ and $v(x) = \cos x$ are linearly dependent **(3 marks)**

(ii) Find the Jacobian of polar coordinate r and θ where $x = r\cos\theta$ and $y = r\sin\theta$ **(5 marks)**

(iii) Verify whether the following functions are functionally dependent, and it so find the relation between them $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$ **(6 marks)**

(b) Evaluate $\iint_R e^{x^2+y^2} dydx$ by transforming it to polar coordinate, where R is the semi-circular region bounded by the x-axis and the curve $x^2 + y^2 = 1$ **(8 marks)**

2. (a) Evaluate the triple integrals $\int_1^3 \int_{-1}^1 \int_0^2 (x + 2y - z)dxdydz$ **(6 marks)**

(b) If $F = \left(3x^2 + 6y\right)i - 14yzj + 20xz^2k$, evaluate the line integral $\int_C F \cdot dr$ along the parametric curves $x(t) = t, y(t) = t^2$ and $y(t) = t^3$ from the point $(0, 0, 0)$ to $(1, 1, 1)$ **(6 marks)**

3. Express each of the following function in the form $u(x, y) + iv(x, y)$, where u and v are real:

(a) Z^3 **(3 marks)**

(b) $\dfrac{1}{1-z}$ **(3 marks)**

(c) e^{3z} **(3 marks)**

(d) $\ln z$ **(3 marks)**

4. (a) The Laplace transforms of $f(t)$ is definition as $L\{f(t)\} = \int_{t=0}^{\infty} f(t)e^{-st}dt$,

show that $L\{\sinh at\} = \dfrac{a}{s^2 - a^2}$ **(8 marks)**

(b) Solve the equation $\dfrac{dx}{dt} - 2x = 4$, given that at $t = 0, x = 1$ by using Laplace transform technique **(4 marks)**

5. For the Fourier series expansion of $f(x)$,

$$f(x) = \frac{a_o}{2} + a_1 \cos x + a_2 \cos 2x + ... + a_n \cos nx + ... \qquad + b_1 \sin x + b_2 \sin 2x + ... + b_n \sin nx + ...$$

Prove that (i) $a_o = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} f(x)dx$ **(4 marks)**

(ii) $b_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} f(x)\sin nx dx$ **(4 marks)**

(iii) $a_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} f(x)\cos nx dx$ **(4marks)**

6. (a) Show that $u(x, y) = x^3 y - y^3 x$ is an harmonic function and find a conjugate harmonic function $v(x, y)$ of $u(x, y)$ **(8 marks)**

(b) Verify that $\cos z = \cos x \cosh y - i \sin x \sinh y$ **(4 marks)**