NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
October Examination 2019

Course Code: MTH 381
Course Title: Mathematical Methods III
Credit Unit: 3
Time allowed: 3 Hours
Instruction: Answer Question Number One and Any Other Four Questions

1(a). Define the following terms;

   (i) Linearly dependent functions **(2 marks)**

   (ii) linearly independent functions **(2 marks)**

(b). Determine whether or not x^2 and e^{-2x} are linearly dependent functions. **(3 marks)**

(c). (i) Find the Jacobian of $u = xy$, and $v = x^2 + y^2$ with respect to x and y. , **(3 marks)**

   (ii) If $x^2 + y^2 + u^2 - v^2 = f_1$ and $uv + xy = f_2$, prove that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{x^2 - y^2}{u^2 + v^2}$. **(6 marks)**

(d). Evaluate $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x + y + z)\, dy\, dx\, dz$ **(6 marks)**

2(a). Given that Z is the conjugate of a complex number Z.

        show that $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ **(4 marks)**

 (b). prove that $|z_1 z_2| = |z_1||z_2|$ **(4 marks)**

 (c). solve the equation $6z^4 - 25z^3 + 32z^2 + 3z - 10 = 0$ **(4 marks)**

3(a). Explain briefly the followings:

   (i) Cauchy-Riemann equations **(2 marks)**

   (ii) Harmonic function? **(2 marks)**

(b). Determine whether or not cos2Z satisfies the Cauchy-Riemann equations **(8 marks)**

4. If $A = (3x^2 - 6yz)i + (2y + 3xz)j + (1 - 4xyz^2)k$, evaluate $\int_c A.\,dr$ from $(0, 0, 0)$ to

      $(1, 1, 1)$ along the following paths C:

      (a) $x = t, y = t^2, z = t^3$ **(4 marks)**

      (b) the straight lines from $(0, 0, 0)$ to $(0, 0, 1)$, then to $(0, 1, 1)$,

         and then to $(1, 1, 1)$.   **(5 marks)**

      (c) the straight line joining $(0, 0, 0)$ and $(1, 1, 1)$ **(3 marks)**

      (d)

5(a). Express $\dfrac{1+2i}{1-3i}$ in the form $r(\cos\theta + i\,\sin\theta)$ **(5 marks)**

 (b). Convert $12\cos(-60°) + \sin(-60°)$ to the rectangular form   **(4marks)**

c. Evaluate $\int_0^3 \int_0^2 (4 - y^2)dydx$ **(3 marks)**

6(a). Determine the poles and the residue at each pole of the function

      $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ **(6 marks)**

(b). Find the residue of $f(z) = \dfrac{ze^z}{(z-a)^3}$ at its pole   **(6 marks)**