NATIONAL OPEN UNIVERSITY OF NIGERIA
PLOT 91, CADASTRAL ZONE, NNAMDI AZIKIWE EXPRESSWAY, JABI - ABUJA
FACULTY OF SCIENCES

DEPARTMENT OF PURE AND APPLIED SCIENCE

OCT/NOV 2019 EXAMINATIONS

**COURSE CODE:** **PHY 301**
**COURSE TITLE:** **CLASSICAL MECHANICS II**
**CREDIT UNIT:** **3**
**TIME ALLOWED:** **(2½ HRS)**

**INSTRUCTION:** ***Answer question 1 and any other four questions***

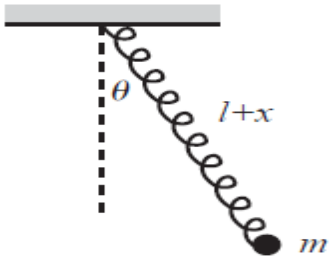
## QUESTION 1

1. (a) Define the generalized force along the generalized coordinate $q_k$. (3 marks)
   (b) What is virtual work? (3 marks)
   (c) State the D'Alembert's principle (3 marks)
   (d) Define the classical Lagrangian. (4 marks)
   (e) A dynamical system with two generalized coordinates $q_1$ and $q_2$ has Lagrangian $L = \dot{q}_1^2 + \dot{q}_2^2$. If $p_1$ and $p_2$ are the corresponding generalized momenta, Determine its Hamiltonian. (6 Marks)

   (f) Briefly explain the term Gravitational Potential (3 Marks)

## QUESTION 2

(a) State the conditions for virtual displacement. (4 marks)
(b) Consider a pendulum made of a spring with a mass m on the end. The spring is arranged to lie in a straight line. The equilibrium length of the spring is $l$. Let the spring have length $l + x(t)$, and let its angle with the vertical be θ(t). Assuming that the motion takes place in a vertical plane, find the equations of motion for x and θ. (8 marks)



**QUESTION 3**

a. (i) What do you understand by constraints? (3 marks)

(ii) Write the constraint equation of the elliptical wire and state if it is scleronomic or rheonomic. (4 marks)

b. Show that velocity dependent constraints are non integrable constraints. (5 marks)

**QUESTION 4**

(a) A one-dimensional harmonic oscillator has Hamiltonian $H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2$. Write down Hamiltonian's equation and find the general solution. (7 marks)

(b) Two particles are connected by a rigid rod so they are constrained to move a fixed distance apart. Write down a constraint equation of the form $f(\vec{r}_1, \vec{r}_2, \dots t) = 0$ and find suitable generalized coordinates for the system incorporating this holonomic constraint. (5 marks)

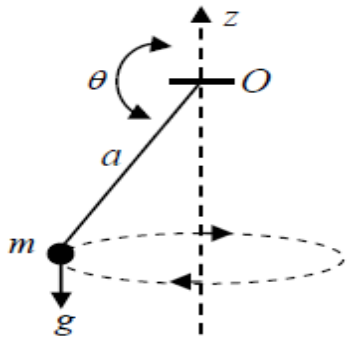
**QUESTION 5**

(a) Consider the motion of a particle in two dimensions given by the Lagrangian

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{\lambda}{4}(x + y)^2$$

Where $\lambda > 0$. The initial conditions are given as $y(0) = 0$, $x(0) = 42$ meters, $\dot{x}(0) = \dot{y}(0) = 0$. What is the value of $x(t) - y(t)$ at $t = 25$ seconds in meters? (6.5 marks)

(b) A particle of mass m is attached to a fixed point O by a weightless inextensible string of length a. It is rotating under the gravity as shown in the figure below. What is the Lagrangian of the particle? (5.5 marks)




**QUESTION 6**

a. State Kepler's second law. (3 marks)

b. Use θ-component of Lagrangian equation to prove Kepler's second law. (9 marks)