NATIONAL OPEN UNIVERSITY OF NIGERIA
PLOT 91, CADASTRAL ZONE, NNAMDI AZIKIWE EXPRESSWAY, JABI - ABUJA
FACULTY OF SCIENCES

DEPARTMENT OF PURE AND APPLIED SCIENCE

2021_2 EXAMINATIONS

COURSE CODE: PHY313
COURSE TITLE: MATHEMATICAL METHODS FOR PHYSICS I
CREDIT UNIT: 3
TIME ALLOWED: (2½ HRS)

INSTRUCTION: *Answer question 1 and any other four questions*

QUESTION 1

a. Suppose that $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

show that $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$ **7 Marks**

b. Evaluate $\displaystyle\int_c \dfrac{z^2 + 1}{z^2 - z}\,dz$ where C is the circle $|z - 1| = 1$ **6 Marks**

c. What is an analytical function? Can a function be differentiable at a point z_0 without being

analytical at z_0 **3 marks**

d. Use Cauchy's integral formula to evaluate $\displaystyle\int_c \dfrac{2z + 1}{z^2 + z}\,dz$ **6 marks**

QUESTION 2

1. a. State two conditions for a function to be analytical **4 marks**

   b. Show that: $\displaystyle\int_0^{\frac{\pi}{2}} e^{t + it}\,dt = \dfrac{1}{2}\left(e^{\frac{\pi}{2}} - 1\right) + \dfrac{i}{2}\left(e^{\frac{\pi}{2}} + 1\right)$ **8 marks**

QUESTION 3

a. Let $w = f(z) = z^2 + 3z$. Find the real part (u) and the imaginary part (v) if of w and calculate
   the value of f at $z = 1 + i3$.   **5 Marks**

b. Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane and find a harmonic
   conjugate function v *of u*   **7 Marks**

QUESTION 4

   Express the following functions in polar form:

   a. $f(z) = z^5 - 4z^2 - 6$ **6 marks**

   b.   State the Cauchy-Riemann equations   **6 marks**

QUESTION 5

   a. Use Cauchy's integral formula, evaluate $\int_c \dfrac{\cos \pi z^2}{(z-1)(z-2)} dz$ where c is $|z|=3/2$ **6 marks**

   b. Explain the term residues and how can it be used for evaluating integrals   **6 marks**

QUESTION 6

a. Given that $u(x, y) = e^{-x} \cos y$, show that $u(x, y)$ is an harmonic function and find the
   function $v(x, y)$ that ensure that $f(z) = u(x, y) + iv(x, y)$ is analytic.   **6 Marks**

b. Evaluate $\int_c \dfrac{z^2 + 1}{z^2 - 1} dz$ where c is the circle $|z+1|=4$ **6 marks**