



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

**FACULTY OF SCIENCES**  
**2021 Examinations**

**Course Code: STT 311**

**Course Title: Probability Distributions II**

**Credit Unit: 3**

**Time Allowed: 3 Hours**

**Total: 70 Marks**

**Instruction: Answer Question One (1) and Any Other 4 Questions**

**Question 1 (Compulsory)**

- (a) State clearly the Chebyshev's Inequality **(4 marks)**
- (b) A sample of 3 balls taken from an urn is examined to determine whether the colour is red (R) or blue (B).
- i. Determine the appropriate sample space for this experiment **(2 marks)**  
Hence, state elements of the
- ii. Event  $E_1$  that exactly one of the balls picked is blue **(2 marks)**
- iii. Event  $E_2$  that the first ball picked is blue **(2 marks)**
- iv. Event  $E_3$  that there are exactly two red balls in the sample **(2 marks)**
- (c) Given that  $x$  is a Bernoulli random variable, state the probability mass function of  $x$  with probability of success as  $p$  and probability of failure as  $(1 - p)$  **(2 marks)**
- (d) Given the following moment generating functions (MGFs), determine the distribution of the random variable  $x$  having the MGFs. Hence or otherwise, find the mean of  $x$ .
- i.  $M_x(t) = e^{8t+5t^2}$  **(2 marks)**
- ii.  $M_x(t) = \frac{13}{13-t}$  **(2 marks)**
- iii.  $M_x(t) = (1 - 5t)^{-6}$  **(2 marks)**
- iv.  $M_x(t) = (0.75 + 0.25e^t)^{48}$  **(2 marks)**

**Question 2**

A semi-boarding school in Abuja has 30% female and 70% male students. The record shows that 20% of the female students and 40% of the male students are boarders while the rest are day students. If a boarding student is picked at random, determine the probability that the student is a male using your idea of Bayes' theorem. **(12 marks)**

### Question 3

A box of compact fluorescent lamps (CFLs) has 10 lamps of which 4 are defective due to vibration during vehicular transit from the factory. If two lamps are selected at random without replacement, find the probability that:

- i. Both lamps are defective (3 marks)
- ii. Both lamps are non-defective (3 marks)
- iii. Only one lamp is defective (3 marks)
- iv. At least one lamp is defective (3 marks)

### Question 4

An experiment involves planting 3 seedlings in a yard. Each of the seedlings has independent chance to survive (S) or not survive (N) while the probability that each seedling will survive is 0.75. Let a random variable  $x$  be the number of surviving trees after one year of planting.

- (a) Construct a probability distribution for this experiment (6 marks)
- (b) Hence, determine the expected value of the random variable  $x$  (6 marks)

### Question 5

The joint density function of the continuous random variables  $x$  and  $y$  is given by

$$f(x, y) = \begin{cases} cxy, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}, \text{ determine}$$

- (a) the value of  $c$  (4 marks)
- (b) the marginal density function of  $x$  (4 marks)
- (c) the marginal density function of  $y$  (4 marks)

### Question 6

The joint probability distribution of the number of taxi cabs and the number of buses that arrive the University terminus park per one-hour interval on a particular Monday morning is given below.

	Number of Taxi Cabs					Total
	0	1	2	3		
0	$4\lambda$	0.09	0.06	$\theta$	$\beta$	
1	0.08	0.03	0.14	0.12	0.37	
2	$\theta$	0.13	0.10	$\lambda$	0.31	
Total	$\alpha$	0.25	0.30	0.20		

- Determine the value of (i)  $\alpha$  (3 marks)
- (ii)  $\beta$  (3 marks)
- (iii)  $\theta$  (3 marks)
- (iv)  $\lambda$  (3 marks)