**NATIONAL OPEN UNIVERSITY OF NIGERIA**
**Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.**

**FACULTY OF SCIENCES**
**April Examination 2019**

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| **Course Code:** | **STT311** |
| **Course Title:** | **Probability Distribution II** |
| **Credit Unit:** | **3** |
| **Time Allowed:** | **3 HOURS** |
| **Total:** | **70 Marks** |
| **Instruction:** | **ATTEMPT QUESTION NUMBER ONE AND ANY OTHER FOUR (4) QUESTIONS** |

1. (a) Give a brief definition of the following terms:
   i. Random experiment
   ii. Sample Space
   iii. Event of a Sample Space
   iv. Probability Measure
   v. Random Variable

   (5 marks)

   (b) Let X be a random variable with probability density function.

$$f(x) = \begin{cases} c(1-x^2)-1, & -1 < x < 1 \\ 0 & elsewhere \end{cases}$$

   What is the value of c?

   (7 marks)

   (c) Given the joint probability distribution

$$f\,(x, y, z) = \frac{xyz}{108} \qquad for\ x = 1, 2, 3;\ \ y = 1, 2, 3;\ \ z = 1, 2$$

   find
   i. the joint marginal distribution of X and Y;
   ii. the joint marginal distribution of X and Z;
   iii. the marginal distribution of X;
   iv. the conditional distribution of Z given X = 1 and Y = 2;
   v. the joint conditional distribution of Y and Z given X = 3

   (10 marks)

2. (a) The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given as $f(x) = \begin{cases} \frac{a}{x^2} & x > 10 \\ 0 & x \leq 10 \end{cases}$

   i. Find the value *a*
   ii. Find P(X > 20) 6 Marks

   (b) Compute E(*X*) and Var(*X*), if *X* has a density function given by

$$f(x) = \begin{cases} \frac{1}{4} x e^{-\frac{x}{2}} & x > 0 \\ 0 & elsewhere \end{cases}$$

   (6 marks)

3. (a) The density function of X is given by $f(x) = \begin{cases} a + bx^2 & 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$ and E(X) = 0.6,

i. Find $a$ and $b$.

ii. Find $E(X^2)$

(4 marks)

(b) The lifetime in hours of an electronic tube is a random variable having a probability density function given by $f(x) = \begin{cases} xe^{-x} & x \ge 0 \\ 0 & elsewhere \end{cases}$.

Compute the expected value lifetime of such a tube

(2 marks)

(c) Derive the moment generating function for a discrete random variable X with the following density function:

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad x = 0,1,2, ....$$

(6 marks)

4. (a) Given that events A and B are independent and that P(A|B) = 0.2 and P(B|A) = 0.5. Compute the probability P(A ∪ B).

(2 marks)

(b) i. If two events, A and B, are such that P(A) = 0.5, P(B) = 0.3, and $P(A \cap B) = 0.1$ Find $P(B|A)$

ii. You are given $P(A \cup B) = 0.65$ and $P(A \cup B') = 0.85$. Determine $P(A)$

(5 marks)

(c) Given that $X_1$ and $X_2$ are two events such that $P(X_1) = 0.45$, $P(X_1 \cup X_2) = 0.68$. Find $P(X_2)$, when

i. $X_1$ and $X_2$ are mutually exclusive

ii. $X_1$ and $X_2$ are independent.

(5 marks)

5. (a) Determine the value of $k$ for which the function given by

$$f(x, y) = kxy \qquad for\ x = 1,2,3,4; \quad y = 1,2,3,4$$

can serve as a joint probability distribution.

(6 marks)

(b) If the joint probability density of X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{4}(2x+y) & for\ 0 < x < 1, 0 < y < 2 \\ 0 & elsewhere \end{cases}$$

find

i. the marginal density of X;

ii. the conditional density of Y given $X = \frac{1}{4}$.

(6 marks)

6. (a) The table below shows the probability distribution function of a random variable X;

| X | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| P(x) | K | 1/12 | K | 1/2 | 1/12 |

Find;

(i) k (ii) $P(X \leq 2)$ (iii) $P(3 \leq X < 5)$

(6 marks)

(b) Let X be a random variable with probability function

$$f(x) = \begin{cases} \frac{2x}{12}, & x = 1, 2, 3 \\ 0 & elsewhere \end{cases}$$

Calculate

i. E(X)

ii. Var(X)

(6 marks)