NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
November Examination 2018

Course Code: **STT311**
Course Title: **Probability Distribution II**
Credit Unit: **3**
Time Allowed: **3 HOURS**
Instruction: **ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) In a Probability distribution class of 50 students, there are 11 from management sciences, 19 sciences, 14 from Health sciences and 6 from education. One student is selected at random.
   (i) What is the sample space of this experiment?
   (ii) Construct a random variable X for this sample space and find its space
   (iii) Find the probability density function for the random variable X (6 marks)

   (b) The probability density function of a random variable X is given by

   $f(x) = \frac{1}{144}(2x-1)$ for x =1,2,3... 12, what is the cumulative distribution function of X (6 marks)

   (c) Show whether or not the real value function $g: R \rightarrow R$ defined by

   $g(y) = \begin{cases} 1+|y| \, if \, -1 < y < 1 \\ 0 \ldots otherwise \end{cases}$ is a probability density function for some random variable Y

   (10 marks)

2. (a) X is a continuous random variable with density function

$$f(x) = \begin{cases} be^{-bx} & x > 0 \\ 0 & otherwise \end{cases}$$

Where b>0. If M(t) is the moment generating function of X, then what is M(-6b)? (6 marks)

(b) Find the moment generating function of a random variable Y having the probability density function given by

$$f(y) = \begin{cases} e^{-y} & y > 0 \\ 0 & otherwise \end{cases}$$

Find the mean and variance of Y (6 marks)

3. (a) State and prove Chebychev's inequality (6 marks)

(b) The probability density function of a random variable X is

$$f(x) = \begin{cases} 630x^4(1-x)^4 & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

What is the exact value of $P(|X-\mu| \le 2\sigma)$? What is the approximate value of $P(|X-\mu| \le 2\sigma)$ when one uses Chebychev inequality? (6 marks)

4. (a) Define the moment generating function of a random variable X. (2 Marks)

(b) Does moment generating function always exist? Give reason. (4 Marks)

(c) The P.M.F of a distribution is given as $p\left(X = x\right) = \frac{m!}{x!\left(m-x\right)!} p^x q^{m-x}, \forall x = 0,1,2,3,...,m.$ Find the moment generating function of the distribution and use it to obtain the mean. (6 Marks)

5. (a) If Y is a random variable with mean μ and variance σ, show that

(i) $\sigma^2 = E(Y^2) - (\mu)^2$ (ii) $Var(aY+b) = a^2 Var(Y)$ (6 marks)

(b) Let a random variable Y has the density function $f(y) = \frac{2y}{c^2}$ for $0 < y \le c$ and f(y) = 0 otherwise, for what value of c is the variance of Y equal 2? (6 marks)

6. (a) For what value of the constant k, is the real value function $g: R \to R$ given by $g(y) = \frac{k}{1+(y-\theta)^2}$, $-\infty < y < \infty$ where θ is a real parameter a probability density function for the random variable Y? (6 marks)

(b) A lot of 8 TV sets include 3 defective. If 4 of the sets are chosen at random for shipment to a hotel. How many defective sets will they expect? (6 marks)