NATIONAL OPEN UNIVERSITY OF NIGERIA
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FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
October Examination 2019

Course Code:	**STT311**
Course Title:	**Probability Distribution II**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Instruction:	**Answer Question Number One and Any Other Four Questions**

1. (a) In a Probability distribution class of 50 students, there are 11 from management sciences, 19 sciences, 14 from Health sciences and 6 from education. One student is selected at random.
   (i) What is the sample space of this experiment? **(2 marks)**
   (ii) Construct a random variable X for this sample space and find its space **(2 marks)**
   (iii) Find the probability density function for the random variable X **(2 marks)**

   (b) The probability density function of a random variable X is given by

$$f(x) = \frac{1}{144}(2x-1) \quad \text{for } x = 1,2,3 \ldots 12,$$

   what is the cumulative distribution function of X **(6 marks)**

   (c) Show whether or not the real value function $g : R \to R$ defined by

$$g(y) = \begin{cases} 1+|y| \; if \; -1 < y < 1 \\ 0 \ldots otherwise \end{cases}$$

   is a probability density function for some random variable Y **(10 marks)**

2. (a) X is a continuous random variable with density function

$$f(x) = \begin{cases} be^{-bx} & x > 0 \\ 0 & otherwise \end{cases} \qquad \text{where } b > 0.$$ **(6 marks)**

   If $M(t)$ is the moment generating function of X, then what is $M(-6b)$?

(b) Find the moment generating function of a random variable Y having the probability density function given by

$$f(y) = \begin{cases} e^{-y} & y > 0 \\ 0 & otherwise \end{cases}$$

Find the mean and variance of Y **(6 marks)**

3. (a) State and prove Chebychev's inequality **(2 marks)**

(b) The probability density function of a random variable X is

$$f(x) = \begin{cases} 630x^4(1-x)^4 & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

What is the exact value of $P(|X-\mu| \le 2\sigma)$? What is the approximate value of $P(|X-\mu| \le 2\sigma)$ when one uses Chebychev inequality? **(10 marks)**

4. (a) Define the moment generating function of a random variable X. **(2 marks)**

(b) Does moment generating function always exist? Give reason. **(4 marks)**

(c) The P.M.F of a distribution is given as $p\left(X = x\right) = \dfrac{m!}{x!\left(m-x\right)!} p^x q^{m-x}, \forall x = 0,1,2,3,...,m.$

Find the moment generating function of the distribution and use it to obtain the mean. **(6 marks)**

5. (a) If Y is a random variable with mean μ and variance σ, show that

(i) $\sigma^2 = E(Y^2) - (\mu)^2$ **(3 marks)**

(ii) $Var(aY + b) = a^2 Var(Y)$ **(3 marks)**

(b) Let a random variable Y has the density function $f(y) = \dfrac{2y}{c^2}$ for $0 < y \le c$ and f(y) = 0 otherwise, for what value of c is the variance of Y equal 2? **(6 marks)**

6. (a) For what value of the constant k, is the real value function $g: R \to R$ given by

$$g(y) = \frac{k}{1 + (y-\theta)^2}, \quad -\infty < y < \infty \text{ where } \theta \text{ is a real parameter,}$$

a probability density function for the random variable Y? **(6 marks)**

(b) A lot of 8 TV sets include 3 defective. If 4 of the sets are chosen at random for shipment to a hotel. How many defective sets will they expect? **(6 marks)**