NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations

Course Code:	**MTH301**
Course Title:	**Functional Analysis I**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One (1) and Any Other 4 Questions**

(1) (a) Define the followings:

(i) Convergent Sequence in a metric space **(2 Marks)**

(ii) closed, open, half open and infinite intervals on the real line. **(4 Marks)**

(b) Let (X,d) be a metric space. Prove that a subset A of X is closed in (X, d) if and only if every convergent sequence of points in A, converges to a point in A. In particular, prove that A is closed in (X, d) if and only if $a_{n \to x,}$ where $x \in \ X \, and \, a_n$ is a sequence of points in A, for all n, implies that $x \in A$. **(8 Marks)**

(c) let (X, d) and $(Y, \ d_1)$ be metric spaces and f a mapping of X into Y. Let $\tau \, and \, \tau_1$ be the topologies determined by d and d_1 respectively. Prove that $f: (x, \tau) \to (y, \tau_1)$ is continuous if and only if $x_n \to x \to f(x_n) \to f(x)$. That is if $x_1, x_2, \dots x_n$ ... is a sequence of points in (X, d) converging to x, then the sequence of points $f(x_1), f(x_2), .. f(x_n) .. in \, (Y, d_1)$ converges to x. **(8 Marks)**

(2) (a) Define the followings:

(i) a metricable topological space **(2 Marks)**

(ii) Euclidean space **(2 Marks)**

(iii) Absolute value **(2 Marks)**

(iv) Norm of a vector. **(2 Marks)**

(b) Let $x \in \Re^n$ and $\mathcal{E} > 0$. Prove that the set $B(x, \varepsilon) = \{y \in \Re^n : d(x, y) < E\}$ is open.

(4 Marks)

(3) Define the following

(i) An interior point **(2 Marks)**

(ii) Boundary point **(2 Marks)**

(iii) Closure of a subset of a set **(2 Marks)**

(iv) Accumulation point **(2 Marks)**

(v) Closed set **(2 Marks)**

(vi) Interior of a set**.** **(2 Marks)**

(4) (a) Define the followings

(i) Neighbourhood of a point of a metric space **(2 Marks)**

(ii) Connected topological space **(2 Marks)**

(iii) Open cover for a subset S of a metric space (X, d) **(2 Marks)**

(iv) Compact subset of a metric space. **(2 Marks)**

(b) Write out two examples of compact subsets of a metric space (X, d). **(4 Marks)**

(5) Write out two examples of the neighbourhood of the point p in a metric space.

(3 Marks each)

(b) Let (K, d) be a compact metric space. Prove that every sequence in K has a convergent subsequence. **(6 Marks)**

(6) For any vector $X \in \mathfrak{R}^n$, state the theorems concerning

(i) the norm, for all $x, y \in X, \alpha \in \mathfrak{R}$. **(4 Marks)**

(ii) the metric d, for all $x, y, z \in X \in \mathfrak{R}^n$. **(4 Marks)**

(iii) Inner product, for all $x, y, y_{1,}$ and $y_2 \in X$ and $\alpha \in \mathfrak{R}$. **(4 Marks)**