NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
April Examination 2019

Course Code:	**MTH301**
Course Title:	**Functional Analysis I**
Credit Unit:	**3**
Time Allowed:	**3 HOURS**
Total:	**70 Marks**
Instruction:	**ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) Define a topological space. (4 Marks)
   (b) Give one example each of
   (i) Indiscrete topology
   (ii) Discrete topology (7 Marks)
   (iii) Usual topology

   (c) Define separable set. (4 Marks)

   (d) Prove that $\mathbb{Q}^n$ is separable. (7Marks)

2. (a) Define open ball (ε-neighbourhood) (5Marks)
   (b) Let $x \in \mathbb{R}^n$, then show that the set $B(x, \varepsilon)$ is open. (7Marks)

3. (a) Define boundary point (5Marks)
   (b) (i) Define closure of subset S of a set X.
   (ii) Is the closure of S normally denoted by $\bar{S}$ closed or open? Justify (7Marks)

4. (a) When is a map $f: A \to B$ (metric spaces) said to be continuous? (5Marks)

   (b) Prove that if $f: A \to B$ between metric spaces is continuous if and only if $f^{-1}(V)$ is open set in A whenever

   V is open set in B. (7Marks)

5. (a) When is a sequence of points x_n in a metric space (X, d) said to be convergent to a point $x \in X$.

   (5Marks)

   (b) Let (X, d) be a metric space. Prove that A of X is closed in (X, d) if and only if every convergent sequence of points in A converges to a point in A. (7Marks)

6. Let X be a metric space and let Y be a subspace of X then prove
   (a) If X is compact and Y is closed in X, then Y is compact . (7Marks)
   (b) If Y is compact, then it is closed in X. (5Marks)