NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS

October Examination 2019

Course Code: **MTH 301**
Course Title: **Functional Analysis I**
Credit Unit: **3**
Time Allowed: **3 Hours**
Instruction: **Answer Question Number One and Any Other Four Questions**

1. (a) Define the following terms:

(i) Let X be a topological space. When is X said to be connected? **(4 marks)**

(ii) Compact set **(3 marks)**

(iii) Sequentially compact set. **(3 marks)**

(b) Give an example of when a topological space X is not connected. **(4 marks)**

(c) Let (X, d) be a metric space. Show that a subset A of X is closed in (X, d) if and only if every convergent sequence of points in A converges to a point in A. (In order words, A is closed in (X, d) if and only if $a_n \to x$ where x ∈ X and a_n is a sequence of points in A ∀n implies that x ∈A). **(8 marks)**

2. (a) (i) Define a metric space X. **(4 marks)**

(ii) Let X be a metric space. Define a ball B of radius r around a point x ∈ X. **(4 marks)**

(b) Give two examples of metric spaces with metric defined. **(4 marks)**

3. (a) Define a topology τ on a set X consists of subsets of X. **(3 marks)**

(b) Prove that a set C in a topological space is a closed set if and only if it contains all its limit points. **(9 marks)**

4. (a) State Heine-Borel theorem. **(2 marks)**

(b) State axioms of addition of a real number system $(\Re, +, \cdot)$. **(3 marks)**

(c) Let (X, d) and (Y, d_1) be metric spaces and f is a mapping of X into Y. Let τ and τ_1 be the topologies determined by d and d_1 respectively.

Show that f(X, τ) →(Y, τ) is continuous if and only if $x_n \to x \Rightarrow f(x_n) \to f(x)$: that is if $x_1, x_2, \ldots x_n, \ldots$ is a sequence of points in (X, d) converging to x, then the sequence of points $f(x_1), f(x_2), \ldots f(x_n), \ldots$ in (Y, d_1) converges to f(x). **(7 marks)**

5. (a) Define a separated set $T \subset S$, where T is a subspace of a topological space S. **(4 marks)**

(b) Prove that a subspace T of a topological space S is disconnected if and only if it is separated by some open subsets U, V of S. **(8 marks)**

6. State the axioms of multiplication and order axiom of a real number system $(\Re, +, \cdot)$. **(12 marks)**