



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**University Village Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja**

**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
**2021\_2 Examinations**

**Course Code: MTH304**  
**Course Title: COMPLEX ANALYSIS**  
**Time Allowed: 3 Hours**  
**Total: 70 Marks**  
**Instruction: Answer Question One (1) and Any Other 4 Questions**

1a. Suppose the function  $f$  given by  $f(z) = u(x, y) + iv(x, y)$  has a derivative at  $z = z_0 = (x_0, y_0)$ . Derive Cauchy Riemann Equations. **(8 marks)**

1b. find all points at  $z = x + iy$  which the function  $f$  given by  $f(z) = x^3 - i(1 - y)^3$  is differentiable. **(8 marks)**

1c. If  $z = x + jy$ , find the locus defined as  $|z| = 5$ . **(6 marks)**

2a. If  $z = \frac{1}{2-j3} + \frac{1}{1-j2}$  express  $z$  in terms of  $a + jb$  **(4 marks)**

2b. Write in polar form  $re^{i\theta}$  (i)  $3 + 4j$  (ii)  $12 + 5j$  **(4 marks)**

2c. If  $z = x + iy$ , find the equation of the locus  $\left| \frac{z+1}{z-1} \right| = 2$  **(4 marks)**

3a. What is a vector valued function? **(4 marks)**

3b. Find a polar form of  $(1 + i)(1 + i\sqrt{3})$  **(4 marks)**

3c. Suppose  $f(z) = 2z^2$ . What is  $\lim_{z \rightarrow 0} \frac{f(z) - f(z_0)}{z - z_0}$  **(4 marks)**

4a. (i) What are conjugates? (2 marks)

(ii)  $(u, v) + (a, v) = (x, y)$  (iii)  $(8,1) + (x, y) = (10,1)$  (2 marks)

4b. i. What are the products of the following? (2 marks)

(ii)  $(4 + 3j)(4 - 3j)$  (iii)  $(x - jy)(x + jy)$  (2 marks)

4c. When are two complex numbers said to be equal? (4 marks)

5a. Let  $c$  be the circle  $|z|=4$ . Evaluate the integral  $\int_c \frac{\cos z}{z^2-6z+5} dz$  (4 marks)

5b. Define Cauchy integral formula (4 marks)

5c. if  $z = x + jy$ , find the equation of the locus  $\arg(z^2) = -\frac{\pi}{4}$  (4 marks)

6a. Find the  $\lim_{x \rightarrow 0} \left\{ \frac{\tan x - x}{x^3} \right\}$  (4 marks)

6b. What Is an entire function? (4 marks)

6c. Compute  $\int_C \frac{1}{z^2+4} dz$  over the contour  $C$  shown below (4 marks)

