NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
April/May 2019 Examinations

Course Code:	**MTH304**
Course Title:	**Complex Analysis I**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One (1) and Any Other 4 Questions**

1. (a) Given that $z_1 = a + ib$, $z_1 = c + id$, then

   Prove that (i) $\overline{z_{1+}z_2} = \overline{z}_1 + \overline{z}_2$ **(6 marks)**

   (ii) $|z_1 z_2| = |z_1||z_2|$ **(6 marks)**

   (b) (i) If $x = \cos\theta + i\sin\theta$, $y = \cos\phi + i\sin\phi$, prove that

   $\frac{x-y}{x+y} = i\tan\left(\frac{\theta-\phi}{2}\right)$ **(7 marks)**

   (ii) Find the modulus of the complex number $\frac{1+2i}{1-(1-i)^2}$ **(3 marks)**

2. (a) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic **(5 marks)**

   (b) If $a = \cos\theta + i\sin\ \theta$, prove that $1 + a + a^2 = (1 + 2\cos\theta)(\cos\theta + i\sin\ \theta)$ **(7 marks)**

3. (a) Let $z = x + iy$, find the real and imaginary parts of the following complex functions

   (i) $f(z) = z^2$ (ii) $f(z) = \frac{1}{z}$ **(6 marks)**

   (b) Hence, show that 3(a) (i) and (ii) satisfy the Cauchy-Riemann equations **(6 marks)**

4. (a) Evaluate each of the following using theorems on limits

   (i) $\lim_{z\to 1-i}\left(\frac{z^2+4z+3}{z+1}\right)$ **(3 marks)**

(ii) $\lim_{z\to -2i} \frac{(2z+3)(z-1)}{z^2-2z+4}$ **(3 marks)**

(b) Differentiate the following complex functions from the first principles

(i) $f(z) = z^2 + z$ **(3 marks)**

(ii) $f(z) = \frac{1}{z}$ **(3 marks)**

5. (a) Let $e^{iz} = \cos z + i \sin z$, prove that $\cos z = \frac{e^{iz}+e^{-iz}}{2}$ **(6 marks)**

(b) Express $\frac{(\cos\theta + i\sin\theta)^8}{(\sin\theta + i\cos\theta)^4}$ in the form $(x + iy)$ **(6 marks)**

6. (a) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path $y = x$. **(4 marks)**

(b) Evaluate $\int_0^{2+i} (\bar{z})^2\, dz$ along the real axis from $z = 0$ to $z = 2$ and then along the line parallel to y −axis $z = 2$ to $z = 2 + i$ **(8 marks)**