NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
November 2018 Examinations

Course Code:	**MTH304**
Course Title:	**Complex Analysis I**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One (1) and Any Other 4 Questions**

1. (a) Given that $z_1 = a + ib$, $z_1 = c + id$, then

   Prove that (i) $\overline{z_{1+}z_2} = \overline{z}_1 + \overline{z}_2$ **(6 marks)**

   (ii) $|z_1 z_2| = |z_1||z_2|$ **(6 marks)**

   (b) (i) Given that $z_1 = 2 + i$ and $z_2 = 3 - 2i$, then evaluate $\left|\frac{2z_2 - z_1 - 5 - i}{2z_2 - z_2 + 3 - i}\right|^2$ **(4 marks)**

   (ii) Find the square root of the complex number $5 + 12i$ **(6 marks)**

2. (a) Show that the complex function $f(z) = z^3$ is harmonic **(6 marks)**

   (b) If $z_1 = r_1(\cos\theta_1 + i\sin\ \theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\ \theta_2)$, prove

   (i) $z_1 z_2 = r_1 r_2\{\cos(\theta_1 + \theta_2) + i\sin\,(\theta_1 + \theta_2)\}$ **(3 marks)**

   (ii) $\frac{z_1}{z_2} = \frac{r_1}{r_2}\{\cos(\theta_1 - \theta_2) + i\sin\,(\theta_1 - \theta_2)\}$ **(3 marks)**

3. (a) Let $z = x + iy$, find the real and imaginary parts of the following complex functions

   (i) $f(z) = z^2$ (ii) $f(z) = \frac{1}{z}$ **(6 marks)**

   (b) Hence, show that 3(a) (i) and (ii) satisfy the Cauchy-Riemann equations **(6 marks)**

4. (a) Evaluate each of the following using theorems on limits

(i) $\lim_{z \to 1+i} (z^2 - 5z + 10)$ **(3 marks)**

(ii) $\lim_{z \to -2i} \frac{(2z+3)(z-1)}{z^2-2z+4}$ **(3 marks)**

(b) Differentiate the following complex functions from the first principles

(i) $f(z) = z^2 + z$ **(3 marks)**

(ii) $f(z) = \frac{1}{z}$ **(3 marks)**

5. (a) Let $e^{iz} = \cos z + i \sin z$, prove that (i) $\cos z = \frac{e^{iz}+e^{-iz}}{2}$ **(4 marks)**

(ii) $\sin\ z = \frac{e^{iz}+e^{-iz}}{2}$ **(4 marks)**

(b) Hence, show that $sin^2 z + cos^2 z = 1$ (**4 marks**)

6. (a) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path (i) $y = x$ (ii) $y = x^2$. **(8 marks)**

(b) Evaluate $\int_0^{2+i} \bar{z}\, dz$ along the real axis from $z = 0$ to $z = 2$ and then along the line parallel to y −axis $z = 2$ to $z = 2 + i$ **(4 marks)**