NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
October Examination 2019

Course Code: **MTH 304**
Course Title: **Complex Analysis I**
Credit Unit: **3**
Time allowed: **3 Hours**
Instruction: **Answer Question Number One and Any Other Four Questions**

1. (a) Define a complex number z. **(3 marks)**

   (b) Suppose $w = z^2$, where $w = u(x, y) + iv(x, y)$.

   Find $u(x, y)$ and $v(x, y)$. **(5 marks)**

   (c) Express $(4-7i)(-2+3i)$ in the form $x + iy$. **(4 marks)**

   (d) State the Cauchy integral formula. **(3 marks)**

   (e) Use the Cauchy Riemann equations to show that the function $f(z) = 2x^3 + 2x + i(3x^2 + 2x)$ is analytic. **(7 marks)**

2. (a) Simplify $\dfrac{5+2i}{(1+i)}$ **(6 marks)**

   (b) If $w = z\bar{z}$. Find $u(x, y)$ and $v(x, y)$. **(6 marks)**

3. (a) Define the limit of a function f at the point $z = z_0$. **(4 marks)**

   (b) Suppose $f(z)$ is a complex function. Find the derivative of

   $f(z) = z^2$ at $z = z_0$. **(8marks)**

4. (a) Define each of the following:

(i)a differentiable function $f(z)$ at a point z_0. **(2 marks)**

(ii) an analytic function $f(z)$ at a point z_0. **(2 marks)**

(iii) an entire function $f(z)$. **(2 marks)**

(b) Find the Laurent Series of $f(z)=\dfrac{1}{z(z-1)}$ valid at $0<|z|<1$. **(6 marks)**

5. (a) State the Cauchy integral formula. **(3 marks)**

(b) Evaluate $\int_c \dfrac{\cos z}{z^2-6z+5}dz$ about $|z|=4$. **(9 marks)**

6. (a) Define a convergent complex sequence. **(3 marks)**

(b) Suppose the function f given by $f(z)=u(x,y)+iv(x,y)$ has a derivative at $z=z_0=(x_0,y_0)$. Find $f'(z_0)$. **(9 marks)**