



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
University Village Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja

**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
2021\_2 Examinations..

**Course Code:** MTH311  
**Course Title:** CALCULUS OF SEVERAL VARIABLES  
**Time Allowed:** 3 Hours  
**Total:** 70 Marks  
**Instruction:** Answer Question One (1) and Any Other 4 Questions

1. a. If  $f(x, y) = \frac{xy^2}{x^2+y^2}$  does  $\lim_{x \rightarrow 0, y \rightarrow 0} f(x, y)$  exist? (5 marks)  
b. Find the derivative of  $z = x^2 + 2xy + y^2$  (5 marks)  
c. Define the following functions: (i) Constant function. (ii) Identity function (iii) Modulus function (iv) Square root function. (v) Trigonometric function. (12 marks)
  
2. a. If  $f(x, y) = x^2 + x^3y^2 + y^4$ . Find the partial derivatives of  $f$  with respect to  $x$  and  $y$  and compute the rates of change of the function in  $x$  and  $y$  directions at the point  $(-1, 2)$  (4 marks)  
b. Find the first order partial derivatives of  $f(x, y) = x^4 + 6y^{0.5} - 10$  (4 marks)  
c. Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2-y^2}{x^2+y^2}$  if it exist (4 marks)
  
3. a. Find the first order partial derivatives for  
 $w = x^2y - 10y^2z^3 + 43x - 7 \tan(4y)$  (4 marks)  
b. Define Curl. (4 marks)  
c. Define a polynomial function of two variables. (4 marks)
  
4. a. Given  $u = x^2 + 2y$  where  $x = r \sin(t)$  and  $\sin^2(t)$ , determine the value of  $\frac{\partial u}{\partial r}$  and  $\frac{\partial u}{\partial t}$  using the chain rule. (6 marks)  
b. Define partial derivatives (6 marks)
  
5. a. Find the second partials and the cross partials of the function  
 $z = 2x^3 + 3xy + 2y^2$  (6 marks)  
b. Using implicit differentiation Find

$$\frac{d(x^3+y^3=6xy)}{dx}$$

**(6 marks)**

6. a. Differentiate  $x^2 - 2xy + y^3 = c$

**(4 marks)**

b. Prove that  $\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$

**(4 marks)**