NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
April/May 2019 Examinations

Course Code:	**MTH311**
Course Title:	**Calculus of Several Variables**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One and Any Other 4 Questions**

1a) Let f be a function defined by $f(x, y) = (x^2 + y, xy)$
Find i) $f(2, -3)$ ii) $f(3,2)$ iii) $f(-2, 3)$ iv) $f(-2, -3)$ **(8 marks)**

b) Evaluate i) $\lim_{\substack{x \to 1 \\ y \to 2}} \frac{3x^2 y}{x^2 + y^2 + 5}$ **(3 ½ marks)**

ii) $\lim_{\substack{x \to \infty \\ y \to 3}} \frac{2x - 3}{x^3 + 4y^3}$ **(3 ½ marks)**

c) If $u = (1 - 2xy + y^2)^{-1/2}$ prove that $x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = y^2 u^3$ **(7 marks)**

2 a) If $x = r\cos\theta$ and $y = r\sin\theta$, then evaluate $\frac{\partial(x,y)}{\partial(r,\theta)}$ **(6 marks)**

b) i) State the Clairaut's Theorem **(2 marks)**
ii) Hence, verify the theorem with $F(x, y) = y^2 e^{2x} + \cos 2y$ **(4 marks)**

3) a) Define total derivative of the function $F(x, y, z, \ldots, u)$ **(3 marks)**
b) Find the value of the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$, where $u = x^2 - y^2$, $v = 2x$ and
$x = r\cos\theta, y = r\sin\theta$. **(9 marks)**

4a) Find $f_{xx}, f_{xy}, f_{yx}, f_{yy}$ of the following:

i) $f = 2x^3 - xy^2 - y^4$ **3marks**

ii) $f = 3e^{-xy} - y\cos x$ **3marks**

b) Let $z = f(x, y)$ where $x = e^u \cos v$, $y = e^u \sin v$ show that

$$y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}$$ **6marks**

5) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for each of the following implicit functions:

i) $z^2 - 2x^4yz^3 = 3x^3 - y^2$ **6 marks**

ii) $y\cos(4xz) = 2z^3 - x^2\, sin(2xy)$ **6 marks**

6 a) Compute a second order Taylor Series expansion around the origin of the function

$f(x, y) = e^x \log(1 + y)$ **3 marks**

(b) State the (i) necessary and (ii) sufficient conditions for a maxima or minima of the function $z = f(x, y)$

5marks

c) Hence find the maxima and minima of the function $z = 2x^2 + xy - y^2 + y$

4 marks