NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
October Examination 2019

Course Code: **MTH 311**
Course Title: **Calculus of Several Variables**
Credit Unit: **3**
Time allowed: **3 Hours**
Instruction: **Answer Question Number One and Any Other Four Questions**

1. (a) Define each of the following:
   (i) a real-valued function of two variables. **(1 mark)**
   (ii) an odd function. **(1 mark)**
   (iii) an even function. **(1 mark)**
   (iv) an inflexion point **(1 mark)**

   (b) Find $\lim_{(x,y)\to(0,0)}\left[\frac{3x^2y}{x^2+y^2}\right]$ if it exists? **(8 marks)**

   (c) Suppose the function h is defined by $h(x)=3x^2-7x-5$, find $h(x-2)$ **(4 marks)**

   (d) If $z(x,y)=2x^3+3xy+2y^2$. Find (i) z_x (ii) z_y (iii) z_{xx} (iv) z_{yy}
   (v) z_{xy} (vi) z_{yx} **(6 marks)**

2. (a) Define each of the following:
   (i) a partial function. **(3 marks)**
   (ii) an exact differential **(3 marks)**

   (b) If $u=(1-2xy+y^2)^{-1/2}$, prove that $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=y^2u^3$ **(6 marks)**

3. (a) Suppose $x^3+y^3=6xy$, find $\frac{dy}{dx}$. **(6 marks)**

   (b) If $y=\sin^{-1}x$, find the derivative of y. **(6 marks)**

4. (a) Define a function of two variables. **(3 marks)**

(b) Find $\lim_{(x,y)\to(1,2)}\left[x^2y^3 - x^3y^2 + 3x + 2y\right]$. **(3 marks)**

(c) Find all the second other derivatives for

$f(x,y) = \cos 2x - x^2e^{5y} + 3y^2$. **(6 marks)**

5. (a) Define the Taylor's Series of a real function $f(x,y)$ **(3 marks)**

(b) (i)State the necessary condition for a function $f(x,y)$ to have a maxima or minima. **(1 mark)**

(ii) State the condition for a function $f(x_1, x_2, ..., x_n)$ to have maximum or minimum **(2 marks)**

(c) Find the maximum and minimum of the function

$z(x,y) = x^2 + xy + y^2 - y$ **(6 marks)**

6. (a)Find the maximum and minimum of $f(x,y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$. **(9 marks)**

(b) Given the function $f(x,y,z,t) = x^2 + y^2 + z^2 + t^2 + xyzt^{-3}$, find the partial derivative of f with respect to t. **(3 marks)**