NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations5678

Course Code: MTH341
Course Title: Real Analysis
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

1a) When is a function f said to be an increasing function in an interval? **(4 marks)**

b)Verify Rolle's theorem for the function f defined by $f(x) = x^3$-6$x^2 + 11x - 6$ for all $x \in [1,3]$.**(6 marks)**

c) Show that if f is differentiable in $]a, b[$ and $f'(x) \neq 0$, for all $x \in]a, b[$, then $f'(x)$ retains the same sign, positive or negative, for all $x \in]a, b[$.**(6 marks)**

d) Find the greatest and the least values of the function f defined by
$f(x) = 3x^4 + 2x^3 - 6x^2 + 6x + 1$ in the interval $[0,2]$.**(6 marks)**

2a) If $f(x) = x^2$, defined on the interval $]a, b[$Show that $f'(c)$ exists if and only if $Lf'(c)$, $Rf'(c)$ exists and $Lf'(c) = Rf'(c)$. **(6 marks)**

b) Let a function $f: [0,5] \longrightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} 2x + 1, \text{when } 0 \leq 3 \\ x^2 - 2, \text{when } 3 \leq x \leq 5 \end{cases}$$

Is the function f derivable at $x = 3$?**(6 marks)**

3a) State without prove the Lagranges Mean Value Theorem. **(3 marks)**

b) Verify the hypothesis and conclusion of Lagrange's mean value theorem for the functions:

(i). $f(x) = \frac{1}{x}$ for all $x \in [1,4]$ **(4.5 marks)**

(ii). $f(x) = \log x$ for all $x \in [1, 1 + \frac{1}{e}]$ all $x \in [2,4]$. **(4.5 marks)**

4a) State without prove the Taylor's Theorem with Schlomilch and Roche form of remainder. **(4 marks)**

b) Deduce the two special form of remainders from 4(a). **(9 marks)**

5a) Determine the values of a and b for which $\lim_{x \to 0} \frac{[x(a - \cos x) + b \sin x]}{x^3}$ exists and is equal to $\frac{1}{6}$.**(6 marks)**

b) Evaluate $\lim_{x \to 0^+} \frac{\log \tan 2x}{\log \tan x}$ **(6 marks)**

6a) State without prove the Maclaurin's Theorem with Lagranges form of remainder **(4 marks)**

b) Show that using Maclaurin's theorem, $\cos x \geq 1 - \frac{x^2}{2}$, for all $x \in \mathbb{R}$ **(8 marks)**