NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_1 Examinations

Course Code: MTH341
Course Title: Real analysis
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

1a)When is a function said to be differentiable at a point? **(4 marks)**

b)State without prove the inverse function theorem. **(4 marks)**

c)Verify the hypothesis and conclusion of Lagrange's Mean Value theorem for the function $f(x) = \frac{1}{x}$ for all $x \in [1,4]$.**(7 marks)**

d) Evaluate $\lim_{x\to 4}\left\{\frac{1}{\log(x-3)} - \frac{1}{x-4}\right.$ **(7 marks)**

2a) Prove that a function f defined on an interval I, is derivable at a point c implies it is continuous at the point c.**(6 marks)**

b) Given that f is a one-one continuous function on an open interval I and $J = f(I)$. If f is differentiable at $x_0 \in I$ and if $f'(x_0) \neq 0$, show that f^{-1} is differentiable at $y_0 = f'(x_0) \in J$ and $(f^{-1})'(y_0) = \frac{1}{f'(x_0)}$. **(6 marks)**

3a)State without prove Rolle's Theorem. **(4 marks)**

b) Verify Rolle's Theorem for the function f, defined by

i. $f(x) = x^3 - 6x^2 + 11x - 6$ for all $x \in [1,3]$

ii. $f(x) = (x-a)^m(x-b)^n$ for all $x \in [a,b]$, $m,n \in \mathbb{N}$ **(8 marks)**

4a) State without prove the General Mean Value theorem. **(3 marks)**

bi) Deduce Lagrange's mean value theorem from the generalised mean value theorem**(4.5 marks)**

bii) Deduce Cauchy's mean value theorem from the generalised mean value theorem $f(z) = \frac{1}{z}$

(4.5 marks)

5ai) State without prove Taylor's infinite series expansion of $f(x)$. **(2 marks)**

aii)State without prove Maclaurin's infinite series expansion of $f(x)$.**(2 marks)**

b) Find the Maclaurin's series expaonsion of (i) $\cos x$ (ii) e^x**(8 marks)**

6a) Determine the values of for all a and b for which $\lim_{x\to 0} \frac{[x(a-\cos x)+b\sin x]}{x^3}$ exists and is equal to $\frac{1}{6}$.**(6 marks)**

b) Evaluate $\lim_{x\to 0^+} \frac{\log\tan 2x}{\log\tan x}$ **(6 marks)**