



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
April Examination 2019

Course Code: MTH341
Course Title: Real Analysis II
Credit Unit: 3
Time Allowed: 3 HOURS
Total: 70 Marks
Instruction: ATTEMPT QUESTION NUMBER ONE AND ANY OTHER FOUR (4) QUESTIONS

- 1 (a) State L-Hospital's rule **(2marks)**
(b) Suppose that f and g are two functions differentiable at $a \in \mathbb{R}$. Prove the Product Rule for differentiation, namely that $(fg)'(a) = f(a)g'(a) + (f)'(a)g(a)$ using the Rules for Limits **(6marks)**

(c) Find the value of $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$ **(6marks)**

(d) if $f(x) = e^x + x \cos(x)$ and $g(x) = \sin(x)$ then find the value of $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$. **(8marks)**

- 2 (a) (i) Define a derivative of a function in an interval. **(3marks)**
(ii) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x^2 \cos\left(\frac{1}{x}\right)$ if $x \neq 0$ and $f(0) = 0$. Find the derivative of f at $x = 0$ if it exists. **(4marks)**

(b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \begin{cases} x^4 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ **(5marks)**
Show that $f'(0)$ exist. And hence find it value.

- 3 (a) State and proof Rolle's theorem **(6marks)**
(b) (i) if $f(x) = x^2 - 3x + 2$ and $g(x) = x^3 - x^2 + x - 1$ then find the value of $\lim_{x \rightarrow 1} \frac{f(x)}{g(x)}$. **(3marks)**
(ii) Show that $\sqrt{3}$ is irrational **(3marks)**

- 4 (a) Does there exist a differentiable function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $f'(0) = 0$ but $f'(x) \geq 1$ for $x \neq 0$? **(5marks)**
(b) Write out the Taylor polynomial $P_2(x)$ of order two at $x = 0$ for the function g and give an expression for the remainder $R_2(x)$ in Taylor's formula $g(x) = \sqrt{1+x} = P_2(x) + R_2(x) \quad -1 < x < \infty$. **(7marks)**

5 (a) (i) Let $\{p_n\}$ be a sequence of real numbers. Give the definition of the convergence of this sequence **(6marks)**

(ii) What are intermediate forms?

(b) Find (i) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + 3xy^2 - xy^2}{x^2 + xy}$ (ii) $\lim_{(x,y) \rightarrow (0,0)} \frac{y^7 x^{98} - x^{97} y^8 + x^{105}}{xy^7 + x^8}$ **(6marks)**

6 (a) Verify the Cauchy's mean value theorem for the functions $f(x) = \sin x$ and $g(x) = \cos x$ in the interval $[-\frac{\pi}{2}, 0]$. **(6marks)**

(b) Let the functions f and g be defined by $f(x) = e^x$ and $g(x) = e^{-x}$ for all $x \in [a, b]$. Show that 'c' obtained from the Cauchy's mean value theorem is the arithmetic mean of a and b . **(6marks)**