NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
November Examination 2018

Course Code:	**MTH341**
Course Title:	**Real Analysis II**
Credit Unit:	**3**
Time Allowed:	**3 HOURS**
Total:	**70 Marks**
Instruction:	**ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1 (a) Suppose $f_n : A \to \Re$ is uniformly continuous on A for every $n \in \aleph$ and $f_n \to f$ uniformly convergent on A.

Prove that f is uniformly continuous on A. (6marks)

(b) Does the result in (a) remain true if $f_n \to f$ pointwise instead of uniformly? (3marks)

(c) If $f(x) = |x|$, show that f has no derivative at $x = 0$. (6marks)

(d) Find $x_0 \in (0, \frac{1}{2})$ when the mean value theorem is applied to $f(x) = x(x-1)(x-2)$. (7marks)

2 (a) (i) Define a derivative of a function in an interval. (3marks)

(ii) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = x^2 \cos(\frac{1}{x})$ if $x \neq 0$ and $f(0) = 0$. Find the derivative of

f at $x = 0$ if it $exists.$ (4marks)

(b) Let $f: R \to R$ be defined as $f(x) = \begin{cases} x^4 \sin(\frac{1}{x}) & if \quad x \neq 0 \\ 0 \quad if & x = 0 \end{cases}$ (5marks)

Show that $f'(0)$ exist. And hence find it value.

3 (a) Verify that the result of Rolle's theorem is not true for $f(x) = 2x^{-2}$ on $[-1,1]$. (6marks)

(b) Find values of $x_0 \in [a,b]$ in the mean value theorem when $f(x) = x^k$, $k = 1,2,3.$ (6marks)

4 (a) Does there exist a differentiable function $f:\Re\to\Re$ such that $f^{'}(0)=0$ but $f^{'}(x)\geq 1$ for $x\neq 0$?

(5marks)

(b) Write out the Taylor polynomial $P_2(x)$ of order two at $x=0$ for the function g and give an expression for the remainder $R_2(x)$ in Taylor's formula g(x) = $\sqrt{1+x}=P_2(x)+R_2(x)\quad -1<x<\infty.$ (7marks)

5 (a) Show that the $\lim_{x\to 0}\left[\frac{1+\frac{x}{2}-\sqrt{1+x}}{x^2}\right]$ exists and find its value (6marks)

(b) Find the least and greatest value of the function f defined by

$f(x)=x^4-4x^3-2x^2+12x+1$ in the interval $[-2,5]$. (6marks)

6 (a) Verify the Cauchy's mean value theorem for the functions $f(x)=\sin x$ and $g(x)=\cos x$ in the interval $[-\frac{\Pi}{2},0]$. (6marks)

(b) Let the functions f and g be defined by $f(x)=e^{x}$ and $g(x)=e^{-x}$ for all $x\in[a,b]$. Show that $'c'$ obtained from the Cauchy's mean value theorem is the arithmetic mean of a and b. (6marks)