



**NATIONAL OPEN UNIVERSITY OF NIGERIA**

**Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja**

**FACULTY OF SCIENCES  
DEPARTMENT OF MATHEMATICS  
October Examination 2019**

**Course Code: MTH 341  
Course Title: Real Analysis  
Credit Unit: 3  
Time Allowed: 3 Hours  
Total Marks: 70**

**Instructions: Answer Question Number One and Any Other Four Questions**

**1. (a) State the Rolle's theorem. (3 Marks)**

(b) Verify Rolle's theorem for  $f(x) = x^2(1 - x^2)$ ,  $0 \leq x \leq 1$ . (12 Marks)

(c) Prove that if  $f$  is continuous at  $x = 0$  then  $xf(x)$  is differentiable at  $x = 0$ . (7 Marks)

**2. (a) Show that the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by**

$$f(x) = \begin{cases} x, & x \geq 1 \\ x^2, & x < 1 \end{cases}$$

Is continuous at  $x = 1$  but is not differentiable at  $x = 1$ . (6 Marks)

(b) Prove that if  $f$  and  $g$  are differentiable at  $x = x_0$ , so also is  
(i)  $f - g$  (ii)  $kf$ , where  $k$  is a constant (6 Marks)

**3. (a) State Cauchy's mean value theorem (4 Marks)**

(b) Show that the functions

$$f(x) = x(x - 1)(x - 2)$$

and

$$h(x) = x(x - 2)(x - 3)$$

defined on  $[0, \frac{1}{2}]$  satisfy the conditions of Cauchy's mean value theorem. (8 Marks)

4. (a) (i) Use the mean value theorem to prove that  $e^x > 1 + x + \frac{x^2}{2}$  if  $x > 0$ . **(3 Marks)**  
(ii) Hence or otherwise, estimate  $\sqrt{21}$ . **(6 Marks)**
- (b) Compute the limit of  $\lim_{x \rightarrow \pi} \frac{\sin 3x}{\tan 5x}$  **(3 Marks)**
5. (a) If  $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ ,  $x \neq 0$ ,  $f(0) = 0$ , show that  $f'(0)$  exists and find it. **(4 Marks)**
- (b) (i) Let  $\varphi: \mathbb{R} \rightarrow \mathbb{R}$  be a function defined by  $\varphi(x) = x - \cos x$ , for all  $x \in \mathbb{R}$ . Show that  $\varphi$  is everywhere differentiable in  $\mathbb{R}$  and obtain  $\varphi'(x)$ . **(4 Marks)**  
(ii) Differentiate with respect to  $x$  the function  

$$f(x) = x^3 \sinh 3x \sin e^{3x}$$
 **(4 Marks)**
6. (a) Use the mean-value theorem to show that if  $f'(x) = 0$  for all  $x$  then  $f$  is a constant. **(4 Marks)**
- (b) (i) Verify the mean value theorem for the function  $f(x) = 2x^2 - 7x + 10$ , in the interval  $[2,5]$ . **(4 Marks)**  
(ii) Compute the  $\lim_{x \rightarrow 0} x^\beta \ln x$ , where  $\beta$  is a constant **(4 Marks)**