NATIONAL OPEN UNIVERSITY OF NIGERIA
PLOT 91, CADASTRAL ZONE, NNAMDI AZIKIWE EXPRESSWAY, JABI - ABUJA
FACULTY OF SCIENCES

DEPARTMENT OF PURE AND APPLIED SCIENCE

2021_1 EXAMINATIONS

COURSE CODE: **PHY312**

COURSE TITLE: **MATHEMATICAL METHODS FOR PHYSICS II**
CREDIT UNIT: **3**
TIME ALLOWED: **(2½ HRS)**

INSTRUCTION: ***Answer question 1 and any other four questions***

QUESTION 1

A (i). Show that the set of values 1, Cos x, Cos2x are orthogonal at the interval $-\pi \leq x \leq \pi$. (State any necessary assumption you have used). [7marks]

B. Define the following
i. Complete solution of a PDE [1mark]
ii. Particular solution of a PDE [1mark]
iii. General solution of a PDE [1mark]
C.(i) While solving a partial differential equation using a variable separable method, what general assumption is made regarding the function which depend on two variables (example u(x,t))? [1marks]
(ii). Find the Laplace transform of $F(t) = e^{at}$. Where $t \geq 0$ and "a" is a constant. [5marks]

D. If $u = x + y + z; v = x^3 + y^3 + z^3 \; and \; w = xyz$; find

$J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$ [6marks]

QUESTION 2

A(i).Verify u(x,t) $= e^{-kt} \, sinx$ satisfies the heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t^2}$$ [5marks]

(ii). When can we say a function is periodic? [3marks]

B. Solve the differential equation $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}=u$ using the method of separation of variables.(Assuming that, $u(0,y)=e^{\frac{2}{y}}$) [4marks]

QUESTION 3

A. Solve the equation $\frac{\partial^2 u}{\partial x^2}$= 12$x^2$(t+1) given that at x=0, u=cos2t and $\frac{\partial u}{\partial x}$ = sint

[7marks]

B. Obtain PDE from $w=f(sinx+Cosy)$ [5marks]

QUESTION 4

A(i). Show that the velocity $u=\frac{ay}{x^2+y^2}; v=\frac{ax}{x^2+y^2}; w=0$ associated with the fluid motion is the flow of an incompressible fluid. [7marks]

(ii). State the property of the Kronecker delta function (δ_{mn}) [2marks]

B. Given that $\Phi(r,\theta)=-E_0 rCos\theta[1+\frac{a^3}{r^3}]$, where Φ is electrostatic potential that satisfied the Laplace equation $\nabla^2\theta=0$. Write the associated electric field components for E_r, E_0 and E_φ

[3marks]

QUESTION 5

A. Solve the equation using Laplace transform $\frac{\partial u}{\partial t}=\frac{2\partial^2 u}{\partial x^2}$; where u(0,t)=u(3,t)=0, u(x,0)=10Sin2πx-6Sin4πx. [7marks]

B. What is the Laplace transform of $f(t)=t^2Cosat$ [5marks]

QUESTION 6

A. Find the period of tan x [6marks]

B. Given the function $\emptyset=x^2+yz$ at the point (1, 2, -1), find its rate of change with distance in the direction $\vec{a}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$. [6marks]