



NATIONAL OPEN UNIVERSITY OF NIGERIA  
PLOT 91, CADASTRAL ZONE, NNAMDI AZIKIWE EXPRESSWAY, JABI - ABUJA  
FACULTY OF SCIENCES

DEPARTMENT OF PURE AND APPLIED SCIENCE

OCT/NOV 2019 EXAMINATIONS

**COURSE CODE:** PHY 312

**COURSE TITLE:** MATHEMATICAL METHODS FOR PHYSICS II

**CREDIT UNIT:** 3

**TIME ALLOWED:** (2½ HRS)

**INSTRUCTION:** Answer question 1 and any other four questions

**QUESTION 1**

- a. Differentiate between ordinary differential equation and partial differential equation [4 marks]
- b. Define the following terms as applied to differential equation:
- (i) Order of a differential equation [2 marks]
  - (ii) Degree of a differential equation [2 marks]
  - (iii) Linearity [2 marks]
- c. Hence classify the following ordinary differential equations in to their order and degree.

(i)  $\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = 0$  [2 marks]

(ii)  $\frac{d^3 y}{dx^3} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$  [2 marks]

(iii)  $L \frac{d^2 y}{dt^2} + R \frac{dq}{dt} + \frac{q}{c} = E \sin wt$  [2 marks]

(iv)  $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2 y}{dx^2}$  [2 marks]

(v)  $\frac{d^2 y}{dx^2} + ax = 0$  [2 marks]

d. Solve the equation

$$\frac{\partial^2 u}{\partial x^2} - 7 \frac{\partial^2 u}{\partial x \partial y} + 6 \frac{\partial^2 u}{\partial y^2} = 0$$

[2 marks]

## QUESTION 2

(a) Define any three of the following functions with example(s):

(i) Odd function

(ii) Even function

(iii) orthogonal function

(iv) Periodic function

(v) Fourier series.

[2 marks each]

b.

Solve the equation  $\frac{\partial^2 u}{\partial x^2} = 12x^2(t+1)$

[6 marks]

given that at  $x = 0, u = \cos 2t$  and  $\frac{\partial u}{\partial t} = \sin t$

## QUESTION 3

(a) Derive an expression for Poisson's equation from Gauss's law in linear material medium.

[ 6 marks]

(b) Hence obtain an expression for the Laplace's equation in Cartesian, cylindrical and spherical coordinates.

[ 6 marks]

## QUESTION 4

a) Solve the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} - 5 \frac{\partial^2 u}{\partial x \partial y} + 6 \frac{\partial^2 u}{\partial y^2} = 0 \quad [5 \text{ marks}]$$

b) Use the Laplace transform to solve the problem

$$\frac{\partial u}{\partial t} = 3 \frac{\partial^2 u}{\partial x^2} \quad [7 \text{ marks}]$$

With the boundary conditions  $u(0, t) = u(3, t) = 0, u(x, 0) = 10 \sin 2\pi x - 6 \sin 4\pi x$

## QUESTION 5

(a) If the general periodic function  $f(x)$  is defined in an interval  $-\pi \leq x \leq \pi$ , Write down the Fourier series of  $f(x)$  in  $[-\pi, \pi]$  defined in a trigonometric series. [4 marks]

(b) Using the Fourier series defined above, Obtain the Fourier coefficients  $a_n, b_n$  and  $a_0$

[8 marks]

### QUESTION 6

(a) Show that the Orthogonality of a Legendre polynomial is given as

$$\int_{-1}^1 P_n(x) P_m(x) dx = 0 \quad m \neq n \quad [6 \text{ marks}]$$

(b) Obtain the general solution of  $x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$  [6 marks]