NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations

Course Code:	**MTH302**
Course Title:	**Elementary Differential Equations II**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One (1) and Any Other 4 Questions**

1. (a) Find a Fourier cosine series for $f(x) = e^x$ on $(0, \pi)$. **(12 marks)**

(b) Find a recurrence formula for the power series solution around x = 0 for the nonhomogeneous differential equation $(x^2 + 4)y'' + xy = x + 2$. **(10 marks)**

2. Determine whether $x = 0$ is a regular singular point of the following differential equations

(i) $2x^2y'' + 7x(x+1)y' - 3y = 0$ **(6 marks)**

(ii) $x^3y'' + 2x^2y' + y = 0$ **(6 marks)**

3. (a) Determine whether $x = 0$ is a regular singular point of the following differential equations

$8x^2y'' + 10xy' + (x-1)y = 0$ **(5 marks)**

(b) Verify the orthogonality property for the Sturm-Liouville problem $y'' + \lambda y = 0; y(0) = 0, y(1) = 0$ if the problem has the eigenvalues **(7 marks)**

$\lambda_n = n^2\pi^2$ corresponding to the eigenfunctions $y_n(x) = A_n \sin n\pi x$, $n = 1,2, \ldots$

4. Find a Fourier sine series for $f(x) = \begin{cases} 0 & x \leq 2 \\ 2 & x > 2 \end{cases}$ on (0,3). **(12 marks)**

5. Find a Fourier sine series for $f(x) = e^x$ on $(0, \pi)$. **(12 marks)**

6. Find the first four terms in each portion of the series solution around $x_0 = 0$ for the following differential equation $(x^2 + 1)y'' - 4xy' + 6y = 0$ **(12 marks)**