NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS

2023_1 POP EXAMINATION

Course Code: **MTH 302**
Course Title: **Elementary Differential Equations II**
Credit Unit: **3**
Time allowed: **3 Hours**
Instruction: **Answer Question One and Any Other Three our Questions**

1(a) Define the following terms

(i) Power series **(2 marks)**
(ii) Radius of convergence **(2 marks)**
(iii) Ordinary points **(2 marks)**
(iv) Singular points **(2 marks)**

(b) Solve the equation $y'' + 4y = 0$ near the ordinary point $x = 0$ **(10 marks)**

(c) By using change of variable $x = e^z$ and $x > 0$, obtain solution of the equation

$x^2y'' + \alpha xy' + \beta y = 0$ **(7 marks)**

2(a) Find the series solution of the equation

$2xy'' + (1 + x)y' - 2y = 0$ **(8 marks)**

(b) Solve the equation $y'' + (x - 1)y' - 4(x - 1) = 0$ about the ordinary point $x = 1$ **(7 marks)**

3(a) Define the following terms

(i) Even function
(ii) Odd function

(b) Classify of the following functions into even, odd or neither

(i) $x \sin nx$ **(1 mark)**

(ii) $x \sin x \cos 4x$ **(1 mark)**

(iii) $(2x + 3) \sin 4x$ **(1 mark)**
(iv) $\sin^2 x \cos 3x$
(v) x^3e^x **(1 mark)**

(b) Determine the Fourier series representation of $f(x) = x^2$

in the interval $-\pi \leq x \leq \pi$. **(10 marks)**

4(a). Determine the Fourier series of a function f(x) defined on the interval $-L \leq X \leq L$ **(5 marks)**

(b). Obtain the Fourier series representation of function $f(x) = x + 1$ for $-1 \leq x \leq 1$ **(10 marks)**

5(a). Define the orthogonality conditions of the set of functions $f_0(x), f_1(x), f_2(x), \ldots\ldots f_n(x)$

with respect to a weight function $w(x)$. **(2 marks)**

(b). Determine the Eigen values and Eigen functions of the system.

$u'' + \lambda u = 0, u(0) = 0, \; u(\pi) = 0$ **(6 marks)**

(c). Prove that Eigen functions of the equation are orthogonal in $[0, \pi]$. **(4 marks)**

6(a). Determine for what values of x the series $\sum_r^\infty a_r(x - x_0)^r$

(i) converges **(2 ½ marks)**

(ii) diverges **(2 ½ marks)**

6(b). Solve the differential equation $y'' + y = 0$ near an ordinary point $x = 0$ **(7 marks)**