NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
April /May 2019 Examinations

Course Code: MTH302
Course Title: Elementary Differential Equations
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One and Any Other 4 Questions

1) (a) Solve the Initial Value Problem $y'' - 2y' - 15y = 0, y'(0) = 1, y''(0) = 2$ **13marks**

  (b) Hence solve $y'' - 2y' - 15y = 3\sin 3x$; $y'(0) = 1, y''(0) = 2$ **9marks**

2) Solve the differential equation $(1 - x^2)\frac{d^2y}{dx^2} - 6x\frac{dy}{dx} - 4y = 0$ near the ordinary point $x = 0$

12marks

3) Solve the differential equation $x^2\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$.

  Hence find y when $\alpha = 11$ and $\beta = 16$ **12marks**

4) (a) Obtain the Fourier Cosine series for $f(x) = \frac{1}{2}x^2$, $-\pi < x < \pi$.

    Hence deduce that $\sum_{n\to 0}^{\infty}\frac{(-1)^n}{n^2} = \frac{\pi^2}{12}$ **9marks**

  (b) Suppose that $f(x)$ is odd obtain the general Fourier Sine Series. **3marks**

5) (a) Using ratio test and radius of convergence, state the conditions under which the power series
    $\sum_{n\to 0}^{\infty} A_n(x - x_0)^n + \ .... $ converges, diverges or fails.

    If $\rho = \lim_{n\to\infty}\left|\frac{A_{n+1}}{A_n}\right|$, what happens if L is zero, finite and infinite? **7marks**

  (b) Determine the radius of convergence of each of the following power series and state if it converges
    or diverges:

      i) $\sum_{n\to 0}^{\infty}\frac{1}{3^n}$ ii) $\sum_{n\to 0}^{\infty}\frac{n^2}{5^n}$ **5marks**

6) (a) Prove that $B(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2m-1}\theta \, d\theta$ **8marks**

  (b) Find the value of $B(5,2)$ **4marks**