

NATIONAL OPEN UNIVERSITY OF NIGERIA University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES November, 2018 Examinations

Course Code: Course Title:	MTH302 Elementary Differential Equations
Credit Unit:	3
Time Allowed:	3 Hours
Total:	70 Marks
Instruction:	Answer Question One and Any Other 4 Questions

1) (a) Solve the Initial Value Problem $y'' - 2y' - 15y = 0, y'(0) = 1, y''(0) = 2$	13marks
(b) Hence solve $y'' - 2y' - 15y = 3\sin 3x$; $y'(0) = 1, y''(0) = 2$	9marks

2) Solve the differential equation
$$(1 - x^2)\frac{d^2y}{dx^2} - 6x\frac{dy}{dx} - 4y = 0$$
 near the ordinary point $x = 0$
12marks

3) Solve the differential equation
$$x^2 \frac{d^2 y}{dx^2} + \alpha x \frac{dy}{dx} + \beta y = 0.$$

Hence find y when $\alpha = 11$ and $\beta = 16$ 12marks

- 4) (a) Obtain the Fourier Cosine series for $f(x) = \frac{1}{2}x^2$, $-\pi < x < \pi$. Hence deduce that $\sum_{n \to 0}^{\infty} \frac{(-1)^n}{n^2} = \frac{\pi^2}{12}$ 9marks
 - (b) Suppose that f(x) is odd obtain the general Fourier Sine Series. **3marks**

5) (a) Using ratio test and radius of convergence, state the conditions under which the power series $\sum_{n\to 0}^{\infty} A_n (x - x_0)^n + \dots$ converges, diverges or fails.

If
$$\rho = \lim_{n \to \infty} \left| \frac{A_{n+1}}{A_n} \right|$$
, what happens if *L* is zero, finite and infinite? **7marks**

(b) Determine the radius of convergence of each of the following power series and state if it converges or diverges:

i)
$$\sum_{n \to 0}^{\infty} \frac{1}{3^n}$$
 ii) $\sum_{n \to 0}^{\infty} \frac{n^2}{5^n}$ 5marks

6) (a) Prove that $B(m,n) = 2 \int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2m-1}\theta \, d\theta$

(b) Find the value of B(5,2)

8marks