NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
October Examination 2019

Course Code: **MTH 302**
Course Title: **Elementary Differential Equations II**
Credit Unit: **3**
Time allowed: **3 Hours**
Instruction: **Answer Question Number One and Any Other Four Questions**

1(a). Given a second order ordinary differential equation $p(x)y''(x) + Q(x)y'(x) + R(x)y(x) = 0$, what do you understand by the regular singular point of the equation? **(4 marks)**

(b). Locate and classify all singular points of the equation

$$x^3(x-1)y'' + (x-1)y' + 4xy = 0.$$ **(4 marks)**

(c). Consider the ordinary differential equation

$$2xy'' + (1+x)y' - 2y = 0$$

(i) Identify the regular singular point of the equation **(1 mark)**

(ii) Assume a series solution about its regular singular point and

find the indicial of the equation **(6 marks)**

(iii) Determine the general solution of the equation **(7 marks)**

2(a). Identify the differential equation

$$x^2y'' + \propto xy' + \beta y = 0$$ **(2 marks)**

(b). Transform the equation into a differential equation with constant coefficients by making use of the transformation$x = e^z$or $z = logx,\ x > 0$. **(7 marks)**

(c) Hence, determine the solution of the equation if the roots of the auxiliary equation obtained have

(i) real and distinct roots (ii) equal real roots **(3 marks)**

3(a). Determine the Fourier series of a function f(x) defined on the interval $-L \leq X \leq L$ **(3 marks)**

(b). Obtain the Fourier series representation of function $f(x) = x + 1$ for $-1 \leq x \leq 1$ **(9 marks)**

4 (a). What are even and odd functions? **(2 marks)**

(b). determine whether or not each of the following functions is even or odd or neither even nor odd

(i) $cosh\,(x)$ **(1 mark)**

(ii) $sinh\,(x)$ **(1 mark)**

(iii) $x^2 + sinx$ **(1 mark)**

(iv) $1 + x + 3x^4$ **(1 mark)**

(c). Determine the Fourier series representation of $f(x) = x^2$

in the interval $-\pi \leq x \leq \pi$. **(6 marks)**

5(a). Define the orthogonality conditions of the set of functions $f_0\,(x), f_1(x), f_2(x), \ldots\ldots f_n(x)$

with respect to a weight function $w(x)$. **(2 marks)**

(b). Determine the Eigen values and Eigen functions of the system.

$u'' + \lambda u = 0, u(0) = 0, \; u(\pi) = 0$ **(6 marks)**

(c). Prove that Eigen functions of the equation are orthogonal in $[0, \pi]$. **(4 marks)**

6(a). Determine for what values of x the series $\sum_r^\infty a_r(x - x_0)^r$

(i) converges **(2 ½ marks)**

(ii) diverges **(2 ½ marks)**

6(b). Solve the differential equation $y'' + y = 0$ near an ordinary point $x = 0$ **(7 marks)**