NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi, Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2022_2 Examinations

Course Code: MTH305
Course Title: Complex Analysis II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 3 Questions

Q1 (a) (i) Define a single-valued complex function $w(z)$. **(3 marks)**

(ii) If $z \in C$ such that $z = x + iy$ and $w(x, y) = u(x, y) = u(x, y) + iv(x, y)$ Suppose $f(z) = z^2$, find $u(x, y)$ and $v(x, y)$, **(4 marks)**

(b) Define each of the following:

(i) a continuous function f at a point z_0. **(3 marks)**

(ii) a branch point. **(3 marks)**

(c) (i) Show that the function $u(x, y) = y^3 - 3x^2y$ is harmonic. **(4 marks)**

(ii) Determine the poles and the residues at the poles of $f(z) = \frac{2z+1}{(z+1)(z-2)}$. **(5 marks)**

(d) State the Green's theorem in a plane. **(3 marks)**

Q2 (a) Define a transformation. **(7 marks)**

(b) Given that z is a complex number and $w = f(z)$. Find $\frac{1}{z}$. **(8 marks)**

Q3 (a) Define the limit of a complex function$f(z)$. **(5 marks)**

(b) Suppose $z \in C$. Show that $sin^2 z + cos^2 z = 1$. **(10 marks)**

Q4 (a) Define each of the following:

(i) removable singularities **(3 marks)**

(ii) bounded complex function. **(4 marks)**

(b) Prove that if $f(z) = \frac{sinz}{z}$ then $z = 0$ is a removable singularity. **(8 marks)**

Q5 (a) State the residue theorem. **(5 marks)**

(b) Expand $f(z) = \frac{1}{z-3}$ in a Laurent series valid for $|z| > 3$. **(10 marks)**

Q6 (a) Define an analytic function $f(z)$. **(5 marks)**

(b) Establish that the real and imaginary part of the function $f(z) = z^2 + 5iz + 3 - i$ satisfy the Cauchy Riemann equation and deduce the analyticity of the function. **(10 marks)**