NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
November Examination 2018

Course Code:	**MTH305**
Course Title:	**Complex Analysis II**
Credit Unit:	**3**
Time Allowed:	**3 HOURS**
Total:	**70 Marks**
Instruction:	**ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) Discuss the transformation of a Complex Variable. (10marks)

   (b) Find the values of z for which $\ell^{4z} = 1$. (6marks)

   (c) Prove that $\sin^2 z_0 + \cos^2 z_0 = 1$. (6marks)

2. (a) Describe the Continuity of a complex functions. (3marks)

   (b) Compute the value of $\sin^{-1} 2$ (4marks)

   (c) Show that $\cos^{-1} z = -i \ln\left|z + i\sqrt{1-z^2}\right|$. (5marks)

3. (a) Distinguish between Taylor series and Laurent series of a complex functions. (2marks)

   (b) Compute the Laurent series for $F(z) = (z-3)\sin\dfrac{1}{z+2}$, about $z = -2$. (5marks)

   (c) Expand $Tan\, z$ using Taylor series about $z = \dfrac{\pi}{2}$. (5marks)

4 . (a) State the residue theorem. (2marks)

   (b) Determine the poles and the residues at the poles of the function $\dfrac{2z+1}{z^2 - z - 2}$. (5marks)

   (c) Use the residue theorem to evaluate, $\int_c \dfrac{5z-2}{z(z-1)} dz$, where c is the circle $|z| = 2$. (5marks)

5. (a) Show that, if $F(z)$ is analytic in a simply connected region R, then $\int_a^b F(z)dz$ is independent of the path in R joining any two points a and b in R. (6marks)

(b) If c is the curve $y = x^3 - 3x^2 + 4x - 1$ joining the points $(1,1)$ and $(2,3)$. Show that $\int_c (12z^2 - 4iz)dz$ is independent of the path joining $(1,1)$ and $(2,3)$. (6marks)

6. (a) Discuss the Cauchy integral theorem (formula). (2marks)

(b) Evaluate (i) $\oint \frac{\sin^6 z}{\left(z - \frac{\pi}{6}\right)^3} dz$, $|z| = 1$ (ii) $\oint \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, $|z| = 3$

(10marks)